Population Aging and Retirement Age Policy: Period Balances, Age Rebalances [a]

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ABSTRACT

We analyze the burden of population aging in pay-as-you-go (PAYG) systems from a period perspective in the world from 1950 to 2100. We benefit from using data that represent all stages of the demographic transition and a variety of demographic trajectories, to elucidate the primacy of changes in period population age structures to the equilibrium of PAYG systems. We investigate to what degree the burden of population aging befalls contributors and beneficiaries in different period policy designs of PAYG systems. Also, we propose a framework to investigate the effectiveness of policies that adjust the retirement age based on gains in longevity to counterbalance the effects of population aging. We examine and apply a method introduced by Bayo and Faber (1981) that adjusts the retirement age based on gains in the mean age at death. We also compare the authors' initial adjustment with a new measurement for the age of retirement based on gains in the modal age at death.

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Population aging impacts retirement systems primarily because it changes the relation between beneficiaries and contributors. In the case of retirement systems structured on period financial balances, known as pay-as-you-go (PAYG) systems, sustainability is directly affected by variations in the old age dependency ratio (OADR), that is, the ratio of the population 65 years of age and older (i.e., potential beneficiaries) to the population 20 to 64 years of age (i.e., potential contributors). For example, in 1889, Germany approved the law that implement the world's first national disability and old-age social security system[1], which set the retirement age for old-age pensions at 70 years of age (Stolleis, 2013).[2] By that time, in 1890, Germany's[3] proportion of the total population with 65 years of age and older was 5.1%, and its OADR was 8.5% (Rahlf et al., 2015). One hundred and twenty-five years later, in 2015, these indicators had respectively changed to 21.1% and 34.8% (United Nations, 2017b). Likewise, when the United States approved the Social Security Act of 1935, its proportion of the total population with 65 years of age and older was 6.1%, and its OADR was 10.7% (U.S. Census Bureau, 2016).[4] Eighty years later, in 2015, the same indicators were equal to 14.6% and 24.6% (United Nations, 2017b). Similarly, the world's OADR has grown from 9.9% in 1950 to 12.8% in 2000, and is expected to be 28.3% in 2050, reaching 41.8% in 2100 (United Nations, 2017b).

External demographic or economic factors that impact retirement systems' financial balances and, consequently, their contributions, benefits, or both are denominated "uninsurable risks" (Settergren, 2001, p. 4). Uninsurable risks cannot be avoided by definition, and are pervasive, they exist in every retirement system, private or public, structured on period or cohort financial balances. Nevertheless, retirement systems can and should safeguard against the impact of these risks (Settergren, 2001). Many policies may buffer the burden of population aging in PAYG systems, such as varying contributions, benefits, or both, and changing the normal ages of contribution and retirement.[5] Among the alternative policies, there is an increasing debate among actuaries, demographers, and economists about adjusting the retirement age based on gains in longevity. But since PAYG systems are established primarily on period financial balances, adjusting the retirement age based on gains in longevity, a life cycle characteristic, may not be effective in lessening the impact of population aging if the contribution of mortality to changes in the population age distribution is only moderate. Therefore, PAYG systems should contemplate the role of the rejuvenating effect of deaths in the definition of retirement age policies. Moreover,

[1] Law Concerning Disability and Old-Age Insurance of 22 June 1889. *Gesetz. betr. die Invaliditäts- und Altersversicherung vom 22. Juni 1889* (Stolleis, 2013, p. 74).

[2] Only in 1916 the retirement age was lowered to 65 years of age. *Gesetz betr. Renten in der Invalidenversicherung v. 12. Juni 1916* (Stolleis, 2013, p. 90).

[3] Deutsche Zollverein.

[4] Excludes Alaska and Hawaii.

[5] Policies that automatically adjust benefits to uninsurable risks are "automatic stabilizers" of the type "automation of the first order" (Scherman, 2011, p.18–22). Scherman (2011) defined two other types of automatic stabilizers: a) notional defined contribution (NDC) designs, specifically, PAYG systems where the contributions of each individual determines one's benefits; and b) automation of the second order, which are based on the PAYG system's financial balance itself.

when old-age mortality declines, adjusting the retirement age based on gains in life expectancy, a mean age at death measure, may be less effective than based on gains in the modal age at death.

We use a *stylized demographic model* to analyze the burden of population aging in PAYG systems in the world from 1950 to 2100. In our stylized demographic model, all population (N) between the age of entry into the labor force (L) and the age of entry into retirement (R) works and contributes to the PAYG system; all population older than the age of entry into retirement (R) is retired and receives benefits from the PAYG system; contributions are equal to contribution rates (con) times wages (w); benefits are equal to benefit rates (ben) times wages (w); age of entry into the labor force (L) and age of entry into retirement (R) are initially fixed; contribution rates (con), benefit rates (ben), and wages (w) are not age-specific; and wages (w) do not vary in response to either the labor market dynamics or productivity changes.

We investigate to what degree the burden of population aging befalls contributors and beneficiaries in different period policy designs of PAYG systems. We benefit from using data that represent all stages of the demographic transition and a variety of demographic trajectories, to elucidate the primacy of changes in period population age structures to the equilibrium of PAYG systems. First, we present the main attributes of PAYG systems in three alternative policy designs: two classic and a third proposed by Musgrave (1981). Second, we review different approaches for measuring the old-age threshold or adjusting the retirement age based on gains in longevity, including a method introduced by Bayo and Faber (1981); and present policies of a selected group of countries that adjust the normal retirement age or retirement pensions based on gains in longevity. Third, we detail our methods and assumptions. Fourth, we estimate the distribution of the burden of population aging between contributors and beneficiaries in different policy designs of PAYG systems. Fifth, we assess the change in the retirement age based on gains in longevity, and how much it alleviates the burden of population aging on contributors and beneficiaries. Sixth, we propose a framework to investigate the effectiveness of policies that adjust the retirement age based on gains in longevity. Last, we propose adjusting the retirement age based on gains in the modal age at death and evaluate its effectiveness.

PERIOD BALANCES: PAY-AS-YOU-GO SYSTEMS

Pay-as-you-go (PAYG) systems are based on *period financial balances* and have no funding of assets. At every period, benefits are honored from contributions made in the same period, that is, each period pays for itself. The period financial balance ensues that the OADR must equal the ratio of the contribution rate (con) to the benefit rate (ben) (Fernandes, 1993, p. 18–20, 93–97;

Keyfitz, 1977, p. 262–265; Keyfitz & Gómez de Léon, 1980). Let a be age; and t, time:

$$\int_L^R N(a,t) \cdot con(t) \cdot w\, da = \int_R^\infty N(a,t) \cdot ben(t) \cdot w\, da \tag{1a}$$

$$\frac{con(t)}{ben(t)} = \frac{\displaystyle\int_R^\infty N(a,t)\, da}{\displaystyle\int_L^R N(a,t)\, da} \tag{1b}$$

Altogether, PAYG systems are built on *intergenerational solidarity*, for today's contributors honor the benefits of today's retirees, taking for granted that the benefits of tomorrow's retirees will be honored by tomorrow's contributors (Fernandes, 1993, p. 18–20). In PAYG systems, it is implicitly assumed that *intergenerational transfers* are unlimited and unbreakable (Keyfitz, 1982; Keyfitz, 1985; Keyfitz, 1988; Keyfitz & Gómez de Léon, 1980; Lapkoff, 1991) and that, otherwise, the state will have the power and disposition to impose it (Keyfitz, 1985, p. 29), that is, the "funding mechanism" is the altruism of future generations (Lapkoff, 1991, p. 160).[6] Therefore, the policy designs of PAYG systems reflect the nature of their social *intergenerational contracts* upon which rest their credibility, long-term political viability, and uninterrupted acceptance as fair by both contributors and beneficiaries (Musgrave, 1981, p. 96–98). Traditionally, we can structure PAYG systems upon two classic policy designs. In the one, the benefit rate (*ben*) is fixed and at every period total contributions adjust via the contribution rate (*con*) to the total benefits honored by the system. This policy design is known as defined benefit (DB) pay-as-you-go (PAYG) system,

$$con(t) = \frac{\displaystyle\int_R^\infty N(a,t)\, da}{\displaystyle\int_L^R N(a,t)\, da} \cdot ben(t) \tag{2}$$

In the other, the contribution rate (*con*) is fixed and at every period total benefits adjust via the benefit rate (*ben*) to the total contributions made to the system. This policy design is known as defined contribution (DC) pay-as-you-go (PAYG) system,

$$ben(t) = \frac{\displaystyle\int_L^R N(a,t)\, da}{\displaystyle\int_R^\infty N(a,t)\, da} \cdot con(t) \tag{3}$$

Therefore, different period policy designs imply distinct life cycle perspectives. In DB PAYG systems, the *promise* is if from L to R each individual contributes a proportional share of the total benefits honored by the system, from R until one's death each individual will receive a fixed percentage of the average wage. In DC PAYG systems, the *promise* is if from L to R each individual

6 On the contrary, fully funded retirement systems, also known as actuarial or reserve, are based on *cohort financial balances* and do have funding of assets. For every cohort, benefits are honored from contributions made by the same cohort, that is, each cohort pays for itself. There are no intergenerational contracts or solidarity (Bourgeois-Pichat, 1978; Fernandes, 1993, p. 69–74, 93–97; Keyfitz, 1977, p. 47–48, 262–265; Keyfitz & Gómez de Léon, 1980).

contributes a fixed percentage of one's wage, from R until one's death each individual will receive a proportional share of the total contributions made to the system (Fernandes, 1993, p. 18–20).

Moreover, different policy designs also lead to distinct distributions of the uninsurable risk of population aging. In DB PAYG systems, this risk befalls contributors via rising *con*, and thus per capita benefits ($ben \cdot w$) improve relative to per capita net wages ($[1-con] \cdot w$). In DC PAYG systems, this risk befalls beneficiaries via declining *ben*, and thus per capita benefits deteriorate relative to per capita net wages (Musgrave, 1981, p. 99–104). As a "fair and practicable solution" for the distribution of the risk of population aging, Musgrave (1981, p. 97, 104) proposed a new policy design that holds constant the ratio of per capita benefits to per capita net wages (ϕ) by adjusting both *con* and *ben* at every period. He named it fixed relative position (FRP) pay-as-you-go (PAYG) system,[7]

$$\phi(t) = \frac{ben(t)}{1 - con(t)} \tag{4a}$$

$$con(t) = \frac{\phi \cdot \int_R^\infty N(a,t)\, da}{\int_L^R N(a,t)\, da + \phi \cdot \int_R^\infty N(a,t)\, da} \tag{4b}$$

$$ben(t) = \frac{\phi \cdot \int_L^R N(a,t)\, da}{\int_L^R N(a,t)\, da + \phi \cdot \int_R^\infty N(a,t)\, da} \tag{4c}$$

In FRP PAYG systems, consequently, the life cycle perspective or *promise* is if from L to R each individual contributes a proportional share of the total benefits honored by the system, and if from R until one's death each individual receives a proportional share of the total contributions made to the system, from L until one's death the ratio of per capita benefits to per capita net wages (ϕ) will hold constant. The risk of population aging befalls both contributors via rising *con*, and beneficiaries via declining *ben*, but ϕ neither improves nor deteriorates. Therefore, FRP PAYG systems have more flexibility for *ad hoc* policy changes to *con* or *ben*, observing that ϕ holds constant, favor greater credibility, long-term viability, and acceptance than DB and DC PAYG systems, that is, a stronger foundation for the social intergenerational contract.

AGE REBALANCES: EQUIVALENT RETIREMENT AGES

Ryder (1975) was the first demographer to propose "a new index of old age" based on changing the concept of age from the number of years elapsed since birth to the numbers of years until

[7] Musgrave (1981, p. 97) identified five PAYG policy designs: a) ad hoc provision, which is a loose agreement where at every period voters decide the level of support; b) fixed replacement rate, which is equivalent to DB; c) fixed contribution rate, which is equivalent to DC; d) fixed replacement rate adjusted, which is similar to DB, but the wage base of beneficiaries is adjusted for productivity and wage increases of contributors; and e) fixed relative position (FRP). In our stylized demographic model, there is no difference between the wage bases of beneficiaries and contributors; consequently, fixed replacement rate adjusted is equivalent to DB.

death. Entry into old-age would be determined not by chronological age, but by the age where life expectancy is equal to "[...] some arbitrary length of time, such as 10 years [...]" (Ryder, 1975, p. 15–17).

Siegel (1980) drew attention to Ryder (1975)'s concept of old-age and its "[...] economic, social, legal, and ethical implications [...]" (Siegel, 1980, p. 346), that is, should socioeconomic groups who have higher mortality or morbidity have earlier access to old-age benefits? He observed that demographers generally use chronological classifications to define the limits of old-age, whereas its cultural definitions vary depending upon the longevity of a population (Siegel, 1980, p. 345–346). He later applied Ryder (1975)'s concept while reviewing new measurements of aging in a work on the aspects of the older population in the United States (Siegel & Davidson, 1984).

The first important addition to Ryder (1975)'s view was the independent work of Bayo and Faber (1981). Their motivation was the public interest, debate, and recommendations of federal government commissions for a gradual increase of the United States Social Security normal retirement age. Gains in life expectancy since 1940 would endorse the proposed gradual increase, however there was no foundation to decide what would be an equitable increase. They recognized that it would be unfair or unreasonable to expect that all extra years of life be spent either in work or in retirement, and proposed a method to measure "equivalent retirement ages" based on declines in mortality "which will be equitable to future retirees relative to past or present retirees" (Bayo & Faber, 1981, p. 1). Bayo and Faber (1981)'s measures of equivalent age are built on three pillars: a) which characteristic or combination of characteristics of a person's life determine equivalence; b) at which point in a person's life we should measure equivalence; and c) which base year we should select as a standard. First, they emphasized that any characteristic should be related to the retirement age only, because if the characteristic were dependent on any other provisions (e.g., contributions or benefits designs) it could neutralize deliberate changes to the social security program. Therefore, they proposed two characteristics to measure equivalence: the "expected number of years spent in retirement" as a limiting case, for it assumes that all gains in life expectancy are spent working; and the "ratio of the expected number of years spent in retirement to the expected number of years spent in the labor force" that equitably distributes the gains in life expectancy between working and retirement years (Bayo & Faber, 1981, p. 3). Second, they advocated measuring equivalence at the age of entry into the labor force (L), for it is an equitable approach as it factors in the experience of people who do not survive to retirement. They also considered measuring equivalence at the age of entry into retirement (R), but only because it is a viable approach. Third, they recommended adopting as base the year when social security benefits were first paid (1940 in their United States context) because it acknowledges "that a specific decision to set the retirement age [...] was made when the program started" (Bayo & Faber, 1981, p. 4). Accordingly, Bayo and Faber (1981) presented four measures of equivalent retirement ages (ERA) that express different perspectives of equity. Their equations are based on the life table functions life expectancy at age a ($\mathring{e}_a$) and number of survivors to age a (l_a), and assume that the age of entry into the labor force is 20 years. Table 1 presents Bayo and

Faber (1981) equations, but with L as the age of entry into the labor force, and R as the age of entry into retirement.

Table 1 – Equivalent retirement age (ERA) by point of measurement and characteristic of measurement

Point of measurement	Characteristic of measurement			
	Expected years in retirement		Ratio of expected years in retirement to expected years in work	
Entry into retirement (R)	$\mathring{e}_R$	(5)	$\dfrac{\mathring{e}_R}{R-L}$	(6)
Entry into labor force (L)	$\dfrac{l_R}{l_L} \cdot \mathring{e}_R$	(7)	$\dfrac{l_R / l_L \cdot \mathring{e}_R}{\mathring{e}_L - l_R / l_L \cdot \mathring{e}_R}$	(8)

Source: Adapted from table in Bayo and Faber (1981, p. 4).

Changes in mortality after R influence all four measures of equivalent retirement ages (ERA). Changes in mortality only between L and R influence ERA measured at L and by the ratio of expected years in retirement to expected years in work (Equation 8), but may or may not influence ERA measured at L and by the expected years in retirement (Equation 7). For example, let a change in adult mortality between L and R result in a decline in the life expectancy at age L ($\mathring{e}_L$), but not in a change of the number of survivors to age R (l_R) and, consequently, not in the probability of surviving between L and R (l_R / l_L). Let also old-age mortality after R remain the same and $\mathring{e}_R$ do not change. In this case, the retirement age would remain the same if determined by Equations 5, 6, and 7, even though it would increase if measured by Equation 8. Notwithstanding, in contexts of declines both in adult and old-age mortality, Equation 7 results in the highest ERA, for it allocates both gains in $\mathring{e}_R$ and in the probability of surviving between L and R (l_R / l_L) to more working years; Equation 5 produces the next to highest, because it allocates gains in $\mathring{e}_R$ to more working years, but does not include gains in l_R / l_L; Equation 8 yields the next to lowest, for it distributes both gains in $\mathring{e}_R$ and in l_R / l_L between working and retirement years; and Equation 6 renders the lowest, because it distributes gains in $\mathring{e}_R$ between working and retirement years, but does not include gains in l_R / l_L. Yet when declines in adult mortality after the base year are minimum, Equations 7 and 5 yield close and the highest ERA; and equations 8 and 6 produce adjoining and the lowest ERA.

Kotlikoff (1981) analyzed the economic effects of gains in longevity.[8] His perspective is of gains in longevity that keep "people young for longer periods of time", and not that keep "old people alive for longer periods" (Kotlikoff, 1981, p. 98). These "youthful" gains in longevity expand the consumption of commodities and leisure by individuals, which demands extra income, and thus increased work. He developed stylized economic models under two demographic scenarios: increasing the expected years in work, keeping the expected years in retirement constant

[8] Although published in the same year as Bayo and Faber (1981)'s paper, Kotlikoff (1981)'s work was based on a paper originally presented at a workshop held in June 1979 (McGaugh & Kiesler, 1981, p. xx).

(Equation 5); and keeping the ratio of expected years in retirement to expected years in work constant (Equation 8). For Kotlikoff (1981), gains in longevity accompanied by increases of expected years in work are beneficial to social security systems via higher ratios of workers (i.e., contributors) to retirees (i.e., beneficiaries), guaranteed that institutional changes eliminate incentives to early retirement, such as the implicit taxation of the work of elderly or retirees.

Bayo and Faber (1981) believed that the mortality trends for many subgroups of the old-age population in the United States had not been or would not be thereafter considerably different from those of the total old-age population. That is, mortality differentials between subgroups of the old-age population had not and would not change substantially over time. Therefore, they argued, adjustments to the retirement age from trends in mortality of the total population would be equitable for its subgroups (Bayo & Faber, 1981, p. 6). Nevertheless, McMillen (1984) showed that there were significant differences between equivalent retirement ages (ERA) estimated separately for men, women and the total population. She used the same data as Bayo and Faber (1981) and likewise assumed 20 years as the age of entry into the labor force (L), and 65 years as the age of entry into retirement (R). As an illustration, for the base year 1940, ERA measured by Equation 5 would be, respectively for men, women and the total population, about 71, 75 and 74 years in 2000, and 73, 78 and 77 years in 2050 (McMillen, 1984, p. 7). ERA measured by Equation 8 would be, accordingly, around 70, 73 and 72 years in 2000, and 72, 75 and 74 years in 2050 (McMillen, 1984, p. 10). McMillen (1984) reasoned about the impacts of the selection of the base year on contexts of increasing mortality differentials; specifically, when the base year changes to subsequent years, more of the mortality differential is included in the baseline and, therefore, future differentials in ERA are smaller (McMillen, 1984, p. 8–9). She concluded observing that to comprehend the differences in ERA by sex is relevant not to set distinct retirement ages for men and for women, but to assist retirement age policies.

Castro and Fernandes (1997) estimated Bayo and Faber (1981)'s four measures for Brazil from 1950 to 2050, separately for men and women, and two retirement benefit scenarios: old-age and length of service. They compared the results with the Brazilian Social Security System and with its projected evolution if the then retirement reform proposals were implemented.[9] Castro and Fernandes (1997) assumed the age of entry into the labor force (L) to be 15 years, and the age of entry into retirement (R) to be 65 years for men and 60 years for women in the old-age retirement scenario, and 50 years for men and 45 years for women in the length of service retirement scenario.[10] In the old-age retirement benefit scenario and base year 1950, ERA measured by Equation 5 would be, respectively for men and women, 70 and 66 years in 2000 (i.e., increases of 5 and 6 years), and 75 and 72 years in 2050 (i.e., increases of 10 and 12 years). Equivalent retirement ages (ERA) measured by Equation 8 would be, correspondingly, 70 and 66 years in 2000 (i.e., in-

9 Specifically: a) replace length of service with length of contribution; b) eliminate special length of service requirements for teachers, journalists, and airline crews; c) set the minimum retirement age at 60 years for men and 55 years for women (Castro & Fernandes, 1997, p. 4).

10 Ultimately, length of service retirement after 35 years of service for men and 30 years of service for women, in agreement with the Brazilian social security legislation at that time.

creases of 5 and 6 years), and 74 and 71 years in 2050 (i.e., increases of 9 and 11 years) (Castro & Fernandes, 1997, p. 8–12). In the length of service retirement scenario, none of the ERA measures but one would reach either for men or women the respective base ages of 65 and 60 years of entry into retirement of the old-age retirement scenario; the exception were ERA for women measured by Equation 6 for base year 1950 (60 years in 2020) and for base year 1960 (60 years in 2050) (Castro & Fernandes, 1997, p. 12–15).

Lee and Goldstein (2003, Supplement) analyzed the consequences of gains in life expectancy for the timing of life cycle stages or events. Their benchmark is the "proportional rescaling of the life cycle" in which all life cycle stages or events change in proportion to variations in life expectancy (Lee & Goldstein, 2003, Supplement, p. 183). Proportional rescaling has two forms: one is "strong proportionality", which modifies both the average and the distribution of timing of life cycle events or stages; the other is "weak proportionality" where only the mean timing of life cycle events or stages change, while their distribution (i.e., variance) does not (Lee & Goldstein, 2003, Supplement, p. 184). Moreover, proportional rescaling can be "flow constrained" where rate or flow variables (e.g., income) are constant and stock variables (e.g., life cycle income) adjust, or "stock constrained" where stock variables (e.g., completed fertility) do not change and flow variables (e.g., fertility rates) adjust (Lee & Goldstein, 2003, Supplement, p. 185).[11] Lee and Goldstein (2003, Supplement, p. 188–190) observed that historical gains in life expectancy have not been distributed equally along the life cycle (see also Horiuchi (1999)), and thus are inconsistent with proportional rescaling. They also emphasized that the correct reference to rescale retirement ages over the life cycle is not the expected years in retirement (Equation 5), but the ratio of expected years in retirement to expected years in work (Equation 8) (Lee & Goldstein, 2003, Supplement, p. 198, p. 204 note 10).

Sanderson and Scherbov (2013) proposed the formal structure for a methodology to measure population aging which translates the values of population characteristics into "alpha-ages". They name this methodology "the characteristic approach". This general and unifying framework was based on their previous studies (Lutz, Sanderson, & Scherbov, 2008; Sanderson & Scherbov, 2005; Sanderson & Scherbov, 2007; Sanderson & Scherbov, 2008; Sanderson & Scherbov, 2010), which extended Ryder (1975) and Lee and Goldstein (2003, Supplement) independently of Bayo and Faber (1981) and Kotlikoff (1981), and was further explored and developed in Sanderson and Scherbov (2014), Sanderson and Scherbov (2015), Sanderson and Scherbov (2017). Alpha-ages with remaining life expectancy as characteristic are named "prospective ages", and measures based on prospective ages are "prospective measures" (e.g., prospective old age dependency ratio (POADR)). Sanderson and Scherbov estimated alpha-ages based on several characteristics, such as remaining life expectancy (Equation 5), ratio of expected years in retirement to expected years in work (Equation 8), and probability of surviving for the next five years; and measures based on alpha-ages that included median age, OADR, and proportions old.

11 In Lee and Goldstein (2003, Supplement)'s framework, Bayo and Faber (1981)'s equivalent retirement ages (ERA) are weak proportional rescaling of the age of entry into retirement, which is analog to Figure 32 in Appendix — Age rebalances: equivalent retirement ages.

In Table 2, we summarize most of the authors we referenced previously, their concepts or methodologies and longevity criteria for adjusting the age of entry into retirement (*R*).

Table 2 – Adjusting the age of entry into retirement (*R*) based on gains in longevity: authors, years, concepts or methodologies, and longevity criteria

Author	Year	Concept / Methodology	Longevity criteria
Ryder (1975)	1975	New index of old-age	Age where life expectancy is equal to some arbitrary length of time
Bayo and Faber (1981)	1981	Equivalent retirement ages	a) expected years in retirement; b) ratio of expected years in retirement to expected years in work. (1)
Kotlikoff (1981)	1981 (2)	Youthful gains in longevity	a) expected years in retirement; b) ratio of expected years in retirement to expected years in work.
Lee and Goldstein (2003, Supplement)	2003	Proportional rescaling of the life cycle	a) ratio of expected years in retirement to expected years in work.
Sanderson and Scherbov (2005) (3)	2005	Characteristic approach (4)	a) expected years in retirement; b) ratio of expected years in retirement to expected years in work (5).

Source: Authors' creation, based on the listed references.
(1): Both measured at either the age of entry into the labor force or the age of entry into retirement.
(2): Based on a paper originally presented at a workshop held in June 1979 (McGaugh & Kiesler, 1981, p. xx).
(3): Further explored and developed in Lutz, Sanderson, and Scherbov (2008), Sanderson and Scherbov (2007), Sanderson and Scherbov (2008), Sanderson and Scherbov (2010), Sanderson and Scherbov (2013), Sanderson and Scherbov (2014), Sanderson and Scherbov (2015), Sanderson and Scherbov (2017).
(4): Named in Sanderson and Scherbov (2013).
(5): Developed in Sanderson and Scherbov (2014).

Equivalent retirement ages: policies of a selected group of countries

Some countries have already implemented or plan to implement policies that adjust the normal retirement age or retirement pensions based on gains in life expectancy. A few of these policies may explicitly mention and establish a minimum age for full retirement, whereas others may do so indirectly and let policyholders chose to retire at the same age but with reduced pensions, that is, partial retirement. This flexibility may indicate that in most cases politicians avoid debating age limits of social programs and, therefore, do not expressly emphasize or enact statutory retirement ages (Scherman, 2011).

In Brazil, length of contribution retirement pensions require a minimum of 35 years of contribution for men, and 30 years for women. Policyholders may retire based on the 85/95 formula or on the *social security factor*. The 85/95 formula refers to the sum of years of contribution and age; originally, if it were at least 85 for women and 95 for men, policyholders were entitled for full length of contribution pensions. The 85/95 formula increased to 86/96 for the biennium 2019/2020, and will continue to increase up to 90/100 in year 2027. If the sum of years of contribution and age is less than the 85/95 formula, the social security factor shall be applied to retirement pensions. The social security factor is based on age, length of contribution, a contribution index equal to 0.31, and life expectancy at the age of entry into retirement (Brasil, 2017; Brasil, 2019; OECD, 2015, p. 222–224). For example, as of April 2019 the social security factor could be between 0.187 (for 15 years of contribution, age 43 years and $\mathring{e}_{43}$=36.6) and 2.094 (for 55 years of contribution, age 70 years and $\mathring{e}_{70}$=15.2). Social security factors that are at least equal to 1.0 may be obtained by combining, for example, 47 years of contribution and age 57 years (factor=1.01 and $\mathring{e}_{57}$=24.8), 42 years of contribution and age 60 years (factor=1.005 and $\mathring{e}_{60}$=22.4), 35 years of contribution and age 65 years (factor=1.022 and $\mathring{e}_{65}$=18.7), and 28 years of contribution and age 70 years (factor=1.019 and $\mathring{e}_{70}$=15.2) (Brasil, 2018).

In Finland, since 2010 earnings-related retirement pensions have been adjusting for gains in longevity by the *life expectancy coefficient*, which is calculated for each cohort at age 62, and is determined by increases in life expectancy since 2009 and a yearly discount rate of 2%. By 2060, the life expectancy coefficient is projected to reduce pensions to 79.2% of their pre-reform values. Starting in 2017, the normal retirement age for earnings-related pensions will raise from 63 to 65 years in increments of 3 months every calendar year. After that, it will be adjusted for gains in life expectancy and limited to 2 months per calendar year (OECD, 2015, p. 251–255; OECD, 2017, p. 34).

In Italy, since 1995 earnings-related retirement pensions are calculated from notional account balances that are converted into annuities by the *transformation coefficient*. The transformation coefficient is estimated based on the life expectancy at age of entry into retirement, the probability that the individual will leave a widow or widower, and the life expectancy of the widow or widower at the pensioner's death. Starting in 2010, the transformation coefficient has been adjusting for changes in life expectancy every three years. Since 2013, the normal retirement age has been automatically adjusting based on $\mathring{e}_{65}$ every three years until 2019, and every two years afterwards. By 2019, the normal retirement age will be 67 years both for men and women. As of 2014, policyholders could retire earlier from age 62 if the length of contribution was at least 42 years and 6 months for men, and 41 years and 6 months for women. Lengths of contribution requirements also increase based on life expectancy (Chłoń-Domińczak, Franco, & Palmer, 2012; OECD, 2015, p. 290–294).

In the Netherlands, the normal retirement age for the state basic old-age pension has been gradually increasing from age 65 years and 2 months in 2014, to age 66 in 2018, 67 in 2021, 67 years and 3 months in 2022, and after that it will be adjusted for gains in life expectancy (OECD, 2015, p. 310–312; OECD, 2017, p. 38).

In Norway, since 2011 income retirement pensions are calculated from "pension entitlements" divided by the *life expectancy divisor*, which is calculated for each cohort at age 61 and based essentially on the remaining life expectancy. Each cohort has different life expectancy divisors from age 62 to age 75. Basic guarantee pensions are adjusted for the life expectancy divisor at age 67 (OECD, 2015, p. 316–319).

In Portugal, the normal retirement age was increased from 65 to 66 years in 2014 and, since 2016, it has been adjusting for gains in longevity, specifically, by the ratio between $\mathring{e}_{65}$ in the first two of the previous three years and $\mathring{e}_{65}$ in the year 2000. The normal retirement age can be reduced by four months for each year of contribution that surpasses 40 years if the individual has reached age 65. Since 2007, earnings-related retirement pensions are the product of reference earnings, an accrual rate, and a *sustainability factor*. The sustainability factor is applied for retirements below the normal retirement age, and is calculated based on gains in $\mathring{e}_{65}$ between the year 2000 and the year before the entry into retirement (OECD, 2015, p. 325–331; OECD, 2019, p. 58–65).

In Sweden, there is no formal retirement age. Policyholders may retire, fully or partially, from the age of 61. Pensions are calculated at the time of retirement by dividing each individual's notional account balance by an *annuity divisor*. The annuity divisor is determined by each cohort's $\mathring{e}_{65}$ and by a discount interest rate of 1.6 percent. Consequently, when $\mathring{e}_{65}$ increases, individuals have to retire later than previous cohorts to receive full pensions; otherwise they receive partial pensions (OECD, 2015, p. 352–353; Scherman, 2011; Settergren, 2001; Settergren, 2003). For example, according to the Annual Report of 2002 of the Swedish Pension System, for the cohort born in 1940, in the year 2005 when it reached age 65, the projected annuity divisor would be 15.7 and $\mathring{e}_{65}$ would be 18 years and 6 months. For the 1965 cohort, in the year 2030 these values would be, respectively, 17.2 and 20 years and 6 months, and its individuals would have to retire 16 months later than the 1940 cohort to have the same proportional pensions. Eventually, for the 1990 cohort, in the year 2055 the projected annuity divisor would be be 18.2 and $\mathring{e}_{65}$ would be 21 years and 11 months, and its individuals would have to retire 26 months later than the 1940 cohort to be entitled for the same proportional pensions (Settergren, 2003, p. 104).

In the Slovak Republic, in 2015 the normal retirement age was 62 with a minimum of 15 years of contribution. Starting in 2017, the normal retirement age would be adjusted for gains in life expectancy. Women with children have reduced normal retirement ages (e.g., in 2014, women with five or more children could retire at 57 years and 6 months), but these retirement ages were increasing and projected to be at least 62 years in 2024 (OECD, 2015, p. 338–341).

In Spain, in 2014 the normal retirement age for full pension was 65 years and two months for those with less than 35 years and 6 months of contribution, and 65 years for those with at least 38 years and 6 months of contribution. Starting in 2019, earnings-related retirement pensions will be adjusted by a *sustainability factor*, which will be determined by the growth in life expectancy of new pensioners. By 2027, the normal retirement age will be 67 years both for men and women (OECD, 2015, p. 348–351).

In the United Kingdom, in 2015 the normal retirement age was 65 years for men and 62 years and 6 months for women, and it was planned to increase to 65 years for women until November 2018. Legislation had been approved to increase the normal retirement age to 66 years by October 2020, and to 67 years between 2026 and 2028. The Government had proposed that later increases in the normal retirement age should be calculated from changes in life expectancy (OECD, 2015, p. 368–371).

In Table 3, we summarize the countries we referenced above, their longevity criteria for adjusting the age of entry into retirement (*R*) or retirement pensions, and respective policies start years.

Table 3 – Adjusting the age of entry into retirement or retirement pensions based on gains in longevity: countries, longevity criteria, and policy start years

Country	Longevity criteria	Policy start year
Brazil	($) Life expectancy at the age of entry into retirement	1999
Finland	(*R*) Increases in life expectancy ($) Increases in life expectancy at age 62	(*R*) 2026 ($) 2010
Italy	(*R*) Life expectancy at age 65. ($) Life expectancy at the age of entry into retirement; probability that the pensioner will leave a widow or widower; life expectancy of the widow or widower at the pensioner's death.	(*R*) 2013, ($) 1995
The Netherlands	(*R*) Increases in life expectancy	2023
Norway	($) Life expectancy at age 61	2011
Portugal	(*R*, $) Increases in life expectancy at age 65	(*R*) 2016, ($) 2007
Sweden	($) Life expectancy at age 65	1998
Slovak Republic	(*R*) Increases in life expectancy	2017
Spain	($) Increases in life expectancy of new pensioners	2019
United Kingdom	(*R*) Increases in life expectancy	2029

Source: Authors' creation, based on Brasil (2017), Brasil (2019), Chłoń-Domińczak, Franco, and Palmer (2012), OECD (2015), OECD (2017), OECD (2019), Scherman (2011), Settergren (2001), Settergren (2003).
(*R*): Age of entry into retirement.
($): Retirement pensions.

DATA

We draw data from the 2017 revision of the official United Nations population estimates and projections (2017 UN REVISION) (United Nations, 2017b; United Nations, 2017c). It covers 150 years from 1950 to 2100, which are divided into two periods: 1950–2015 (estimates) and 2015–2100 (projections), and has nine projection variants. We use the medium fertility projection variant, which combines the medium fertility, normal mortality, and normal international migration assumptions (United Nations, 2017c). The 2017 UN REVISION covers a total of 233 countries and areas. It includes detailed data (e.g., population by five-year age groups) for the 201 countries and areas that had 90,000 or more inhabitants in 2017, and only total populations and growth rates for the remaining 32 (United Nations, 2017c, p. 1). We include these 201 countries and areas, both sexes combined, and the variables: a) list of locations with code, description, region and subregion; b) populations by five-year age groups; c) deaths by five-year age groups; d) abridged life tables by five-year age groups; and e) demographic indicators.

The 2017 UN REVISION (United Nations, 2017a; United Nations, 2017b; United Nations, 2017c, p. vii) follows the names and composition of geographic areas of the United Nations' *Standard Country or Area Codes for Statistical Use (M49)* (United Nations, 2018), but with two differences. First, the 2017 UN REVISION groups its countries and areas into six regions: Africa, Asia, Europe, Latin America and the Caribbean, Northern America, and Oceania; whereas the United Nations (2018) adopts five geographic regions based on continental regions: Africa, Asia, Europe, Americas, and Oceania. Second, while the 2017 UN REVISION combines the Southern Asia and Central Asia subregions into South-Central Asia; the United Nations (2018) classifies Central Asia and Southern Asia as separate subregions since 2005. Yet none of the 2017 UN REVISION's geographic classification criteria help us to either summarize or drill down its data. First, Northern America has no subregions, and only two countries with detailed data (i.e., Canada and United States of America); second, we risk loosing information when we combine subregions. Therefore, we fine-tune the 2017 UN REVISION's regional and subregional classification of countries and areas. First, we adopt the United Nations (2018)'s standard, specifically, five geographic regions, and Central Asia and Southern Asia as separate subregions. Second, we remove Latin America and the Caribbean as a subregion, but maintain its subregions under Americas; that is, we categorize Americas' subregions as the Caribbean, Central America, South America, and Northern America.

Most of our methods work with the open-ended age group, and some also incorporate simultaneous use of distinct variables by age groups (e.g., populations or deaths multiplied by life table functions). We make the following changes to obtain populations and deaths with the same open-ended age groups as life tables (Table 4): a) populations from 1990 to 2100: decrease open-ended age group to 95+ (add 95–99 and 100+); b) life tables from 1950–1955 to 1985–1990: add open-ended age group 80+; and c) life tables from 1950–1955 to 2095–2100: increase open-ended age group to 95+.

Table 4 – Open-ended age groups of the 2017 UN REVISION by variable, year or period, and before and after adjustments

Variable	Years / Periods (1)	Open-ended age group	
		Before	After
Populations	1950 to 1989	80+	80+
Deaths	1950–1955 to 1985–1990	95+	95+
Abridged life tables	1950–1955 to 1985–1990	85+	80+ and 95+
Populations	1990 to 2100	100+	95+
Deaths	1990–1995 to 2095–2100	95+	95+
Abridged life tables	1990–1995 to 2095–2100	85+	95+

Source: Authors' calculations, based on United Nations (2017b).
(1): Annual data refer to 1 July of the year indicated. Data for five-year periods are from 1 July of the first year to 30 June of the final year.

Model old-age mortality

We model old-age age-specific death rates to increase the life tables' open-ended age group to 95+ (Fernandes, 2019). We follow Thatcher, Kannisto, and Vaupel (1998) to choose the explanatory mathematical mortality models; and Horiuchi, Ouellette, Cheung, and Robine (2013) to use the old-age modal age at death (M) as the parameter for the overall level of mortality. We use the following mathematical mortality models: Makeham (Makeham, 1860), and Makeham variants of logistic (Perks, 1932), Kannisto (Kannisto, 1992 as cited in Thatcher et al., 1998, p. 16) and Weibul (Weibull, 1951).[12,13,14] We choose as the final best model for each geographic area and 5-year period, the one that has the minimum arithmetic average absolute relative differences calculated over the oldest five age groups that were used to fit the models in that period.[15]

Demographic Determinants of Population Aging

We decompose the rate of change in the mean age of the population in the world from 1950 to 2100 according to the first mathematical expression from Preston, Himes, and Eggers (1989) (PHE I) (Fernandes, 2019; Fernandes & Turra, 2019).

The PHE I decomposes the rate of change in the mean age of a population into *rejuvenating effects* of births, deaths, in-migration, and out-migration. Let N be population; a, age; t, time; I, in-migrants; D, deaths; O, out-migrants; b, crude birth rate; i, crude in-migration rate; d, crude death rate; o, crude out-migration rate; and $dN_{\overline{a}}(t)/dt$, the first derivative of the mean age of the population ($N_{\overline{a}}$) with respect to time (Preston et al., 1989, p. 695).:

$$\begin{aligned}\frac{dN_{\overline{a}}(t)}{dt} = 1 & \\ & - b(t) \cdot N_{\overline{a}}(t) \\ & - i(t) \cdot [N_{\overline{a}}(t) - I_{\overline{a}}(t)] \\ & - d(t) \cdot [D_{\overline{a}}(t) - N_{\overline{a}}(t)] \\ & - o(t) \cdot [O_{\overline{a}}(t) - N_{\overline{a}}(t)]\end{aligned} \tag{9}$$

12 In all our Makeham variants mathematical mortality models, the modal age at death is from senescent mortality (M_s), which is practically equal to while somewhat higher than the modal age at death (M), assuming that at old ages the proportional level of premature mortality given by the Makeham term is very low (Horiuchi et al., 2013, p. 54).

13 Horiuchi et al. (2013) did not work with or reformulate the Kannisto model in terms of M or M_s. Nevertheless, we derive it as a special case of the logistic model.

14 We employ the R language and environment (`Rstats`) (R Core Team, 2018) with the MortalityLaws `R` package (`MortalityLaws`) (Pascariu & Canudas-Romo, 2017; Pascariu, 2018), and use the `MortalityLaws` feature that let us define our own parametrized mortality functions.

15 60–64 to 80–84 from 1950–1955 to 1985–1990, and 70–74 to 90–94 from 1990–1995 to 2095–2100. We estimate additional life table age-specific death rates for the 85–89 and 90–94 age groups from 1990–1995 to 2095–2100 based on the 2017 UN REVISION data for populations and deaths.

In Equation 9, 1 is the population natural tendency to age one time unit ($dN_{\overline{a}}(t)$) by each one calendar time unit (dt), $b(t) \cdot N_{\overline{a}}(t)$ is the rejuvenating effect of births, $d(t) \cdot [D_{\overline{a}}(t) - N_{\overline{a}}(t)]$ is the rejuvenating effect of deaths, $i(t) \cdot [N_{\overline{a}}(t) - I_{\overline{a}}(t)]$ is the rejuvenating effect of in-migration, and $o(t) \cdot [O_{\overline{a}}(t) - N_{\overline{a}}(t)]$ is the rejuvenating effect of out-migration. That is, the rejuvenating effects of births, deaths, in-migration, and out-migration are the products of their respective relative volumes (i.e., crude rates) and age selectivities (i.e., mean age differences to the mean age of the population).

Since the 2017 UN REVISION does not include migration age schedules, and is limited to net numbers of migrants ($I-O$) and net migration rates ($i-o$), it precludes the estimation of the mean age of migrations. Therefore, we adopt an approach similar to the one used elsewhere (Preston et al., 1989; Preston & Stokes, 2012) for the second mathematical expression from Preston et al. (1989) (PHE II), and compute the rejuvenating effect of net migration as a residual ($\epsilon_{\overline{a}}$), specifically,

$$\epsilon_{\overline{a}}(t) = i(t) \cdot [N_{\overline{a}}(t) - I_{\overline{a}}(t)] + o(t) \cdot [O_{\overline{a}}(t) - N_{\overline{a}}(t)] \tag{10}$$

$$\implies \frac{dN_{\overline{a}}(t)}{dt} = 1 - b(t) \cdot N_{\overline{a}}(t) - d(t) \cdot [D_{\overline{a}}(t) - N_{\overline{a}}(t)] - \epsilon_{\overline{a}}(t) \tag{11}$$

METHODS AND ASSUMPTIONS

We use our stylized demographic model to analyze the burden of population aging on defined benefit (DB), defined contribution (DC), and fixed relative position (FRP) PAYG systems in the world from 1950 to 2100. We adopt 1950–1955 as the base period, 20 years as the age of entry into the labor force (L), and 65 years as the age of entry into retirement (R). Benefit rates (ben) in DB PAYG systems are 100%; contribution rates (con) in DC PAYG systems are 15%; and ratios of per capita benefits to per capita net wages (ϕ) in FRP PAYG systems are equal to 100%.

Equivalent retirement ages (ERA) are for the five-year periods of the 2017 UN REVISION, specifically, in the period 1950–1955 all countries R and ERA are 65 years, then from 1955–1960 to 2095–2100 ERA are determined by the respective five-year period life tables. Still, since we would strictly need to estimate ERA for 2100–2105 to calculate OADR from ERA for 2100 and because the 2017 UN REVISION presents no life tables for 2100–2105, we assume that life tables and, consequently, ERA for 2100–2105 are the same as those for 2095–2100. Likewise, we presume that the rejuvenating effects of births, deaths and migration for 2100–2105 are the same as those for 2095–2100.

We reference equivalent retirement ages (ERA) in terms of the number of person-years lived above age a (T_a) and the number of survivors to age a (l_a) as in Table 5. Also, we follow Bayo and Faber (1981), Kotlikoff (1981), Lee and Goldstein (2003, Supplement), Sanderson and Scherbov (2014), and use for ERA the ratio of expected years in retirement to expected years in work measured at the age of entry into the labor force (L), specifically, we use Equation 15.

Table 5 – Equivalent retirement age (ERA) by point of measurement and characteristic of measurement in terms of T_a and l_a

Point of measurement	Characteristic of measurement			
	Expected years in retirement		Ratio of expected years in retirement to expected years in work	
Entry into retirement (R)	$\frac{T_R}{l_R}$	(12)	$\frac{T_R}{l_R \cdot (R-L)}$	(13)
Entry into labor force (L)	$\frac{T_R}{l_L}$	(14)	$\frac{T_R}{T_L - T_R}$	(15)

Source: Authors' creation, based on table in Bayo and Faber (1981, p. 4).

Last, analog to the "prospective old age dependency ratio (POADR)" term introduced in Sanderson and Scherbov (2007, p. 48), and the terminology "prospective ages" vis-à-vis "prospective measures" in Sanderson and Scherbov (2008) and their later works; we name old age dependency ratios (OADR) calculated from equivalent retirement ages (ERA), equivalent old age dependency ratios (EOADR).

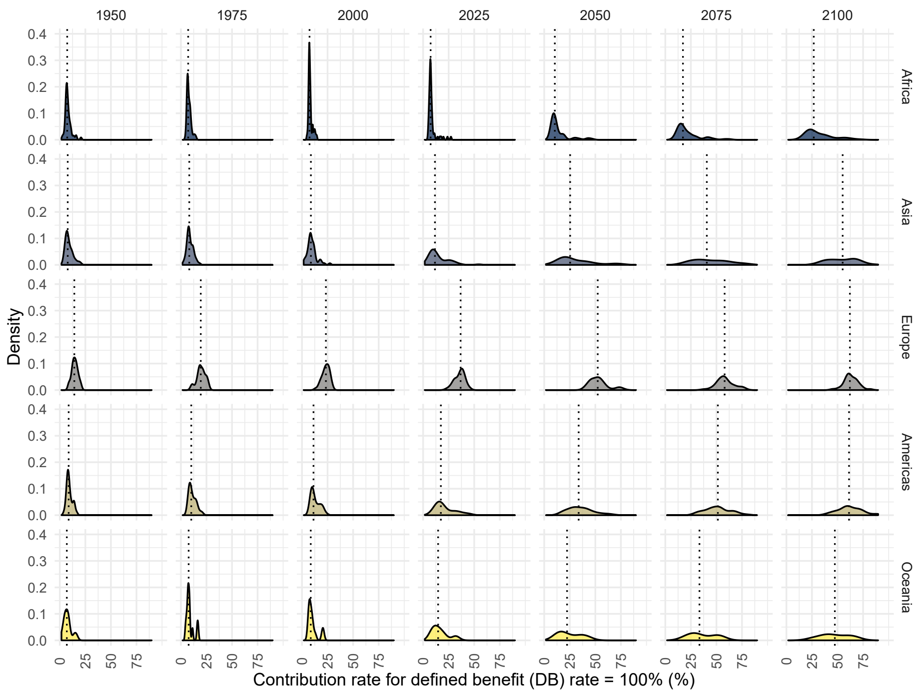
PAY-AS-YOU-GO SYSTEMS: DISTRIBUTION OF THE BURDEN OF WORLD POPULATION AGING

We estimate the distribution of the burden of population aging between contributors, via contribution rates (*con*), and beneficiaries, via benefit rates (*ben*), of defined benefit (DB), defined contribution (DC), and fixed relative position (FRP) PAYG systems. Figures 1 to 4 plot the densities of contribution rates (*con*) and benefit rates (*ben*) of DB, DC, and FRP PAYG systems for selected years and all regions; and Figures 5 to 8 plot their distribution in 2100 by subregions.

Between 1950 and 2100, the median contribution rate (*con*) of DB PAYG systems increases from 7.2% to 26.7% in Africa, from 7.7% to 55.0% in Asia, from 14.2% to 62.0% in Europe, from 8.7% to 61.2% in the Americas, and from 6.9% to 47.4% in Oceania. In 2100, the median *con* of DB PAYG systems is above 60% in as many as nine of the twenty-two subregions: Southern Europe (68.8%), Eastern Asia (66.5%), Southern Asia (66.4%), Western Europe (64.9%), Caribbean (64.7%), Central America (63.4%), Australia/New Zealand (61.5%), Northern Europe (60.0%) and Eastern Europe (59.6%). In the same period, the median benefit rate (*ben*) of DC PAYG systems decreases from 206.9% to 56.1% in Africa, from 193.4% to 27.3% in Asia, from 105.5% to 24.2% in Europe, from 172.7% to 24.5% in the Americas, and from 217.7% to 31.7% in Oceania. In 2100, the median *ben* of DC PAYG systems is below 25% for the same nine subregions: Southern Europe (21.8%), Eastern Asia (22.6%), Southern Asia (22.6%), Western Europe (23.1%), Caribbean (23.2%), Central America (23.8%), Australia/New Zealand (24.4%), Northern Europe (25.0%) and Eastern Europe (25.2%).

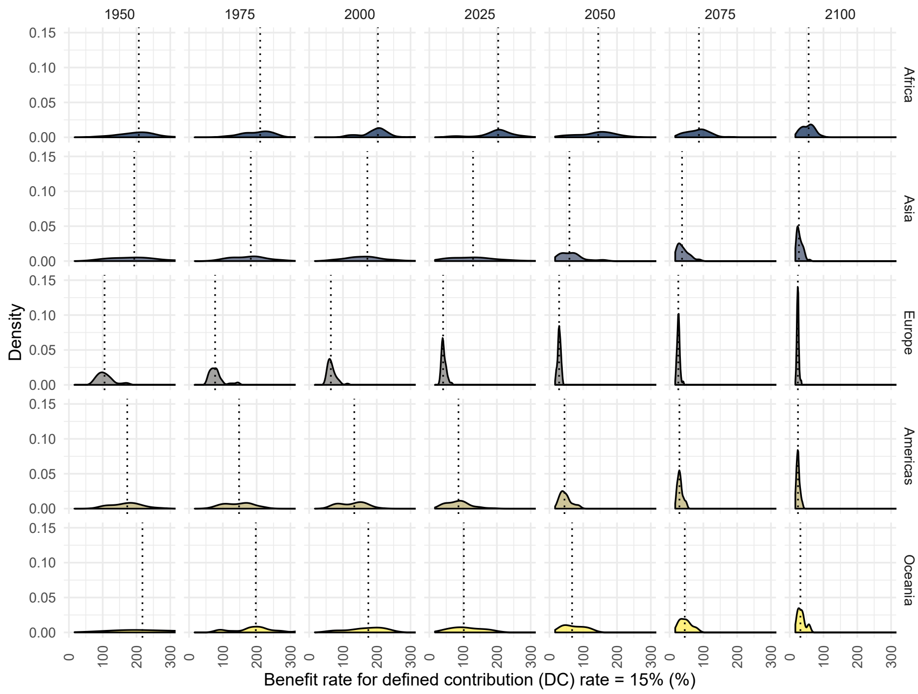
Except for Africa in general, these figures demonstrate the long-term unfeasibility of DB and DC PAYG systems in aging populations. Yet adopting FRP PAYG systems, and thus distributing the risk of population aging between contributors and beneficiaries, may lead to more credible, nevertheless still demanding, scenarios. Between 1950 and 2100, the median contribution rate (*con*) of FRP PAYG systems increases from 6.7% to 21.1% in Africa, from 7.2% to 35.5% in Asia, from 12.4% to 38.3% in Europe, from 8.0% to 38.0% in the Americas, and from 6.4% to 32.1% in Oceania. Still in the same period, the median benefit rate (*ben*) of FRP PAYG systems decreases from 93.2% to 78.9% in Africa, from 92.8% to 64.5% in Asia, from 87.5% to 61.7% in Europe, from 92.0% to 62.0% in the Americas, and from 93.5% to 67.8% in Oceania. In 2100, for the said nine subregions with median *con* of DB PAYG systems above 60% and median *ben* of DC PAYG systems below 25% in FRP PAYG systems, the median *con* is between 40.8% (Southern Europe) and 37.3% (Eastern Europe), and the median *ben* is between 59.2% (Southern Europe) and 62.7% (Eastern Europe).

Figure 1 – Density of the contribution rate (*con*) of DB by selected periods and regions



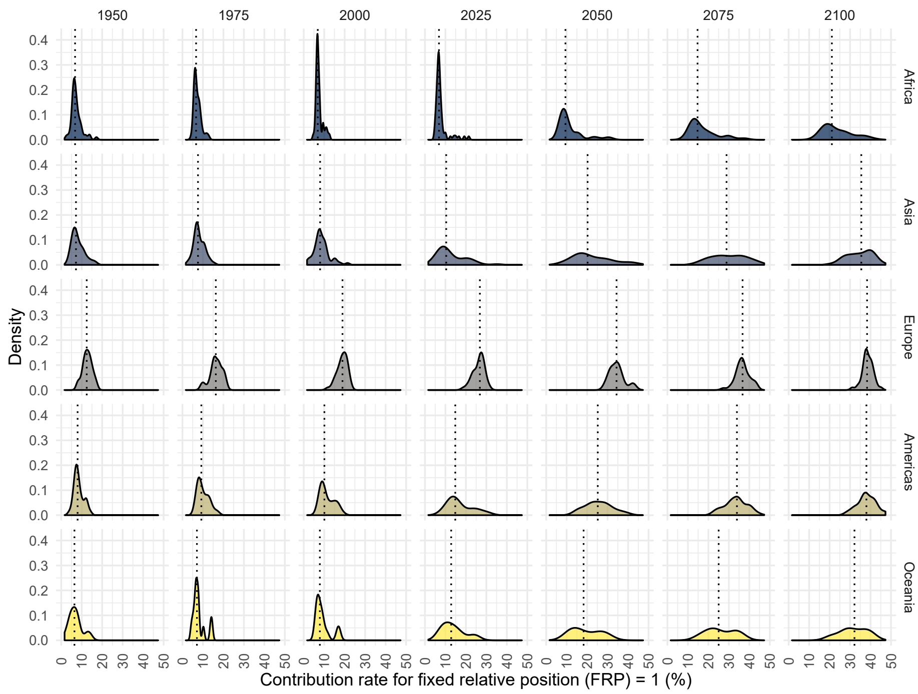
Source: Authors' calculations, based on United Nations (2017b).
Notes: Benefit rate (*ben*) = 100%. Vertical dotted line indicates the median of the distribution.

Figure 2 – Density of the benefit rate (*ben*) of DC by selected periods and regions



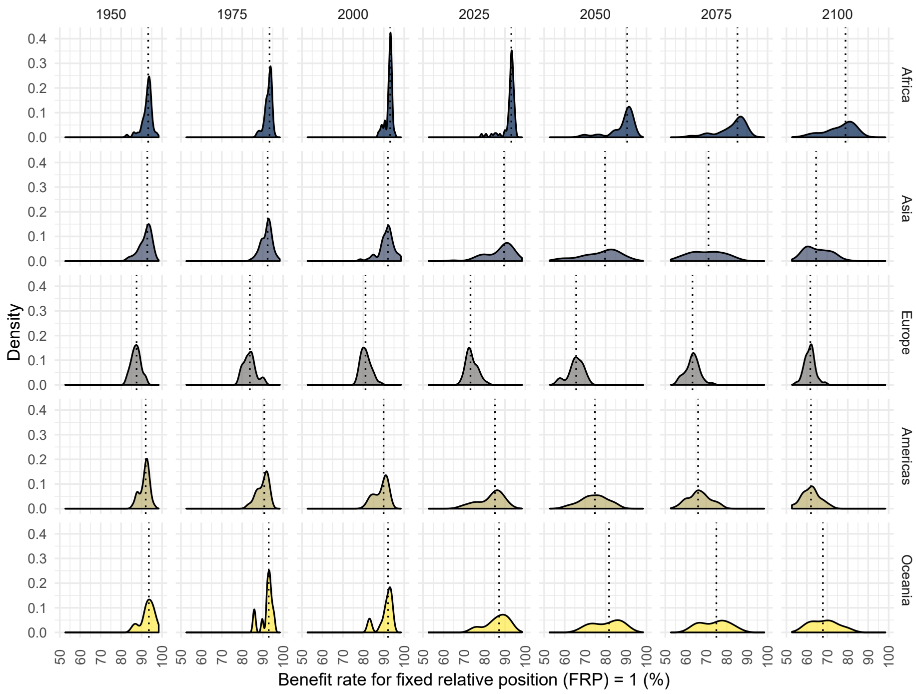
Source: Authors' calculations, based on United Nations (2017b).
Notes: Contribution rate (*con*) = 15%. Vertical dotted line indicates the median of the distribution.

Figure 3 – Density of the contribution rate (*con*) of FRP by selected periods and regions



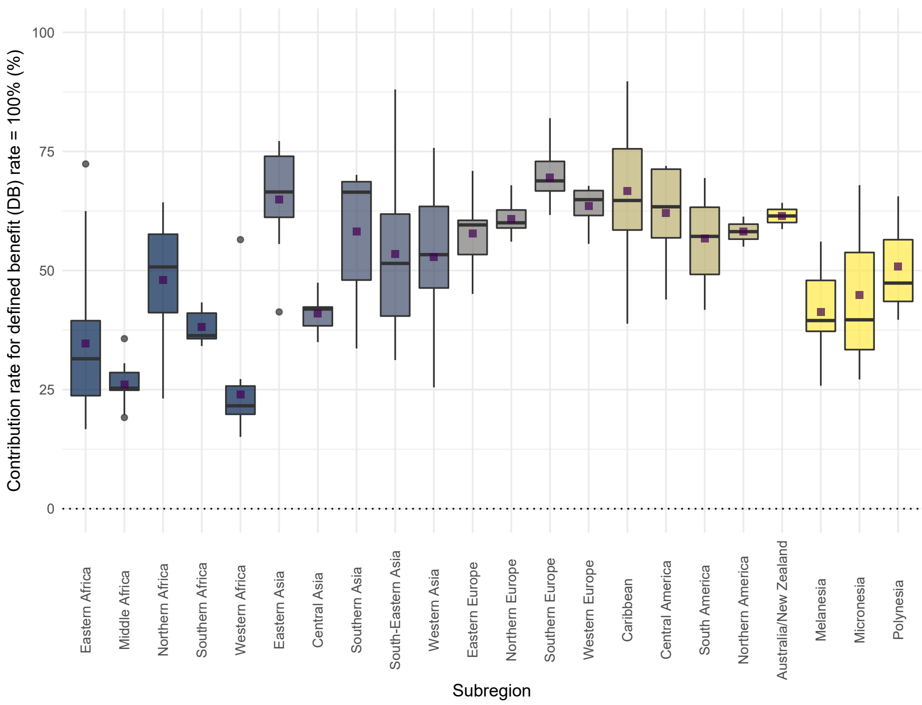
Source: Authors' calculations, based on United Nations (2017b).
Notes: Ratio of per capita benefits to per capita net wages (ϕ) = 100%. Vertical dotted line indicates the median of the distribution.

Figure 4 – Density of the benefit rate (*ben*) of FRP by selected periods and regions



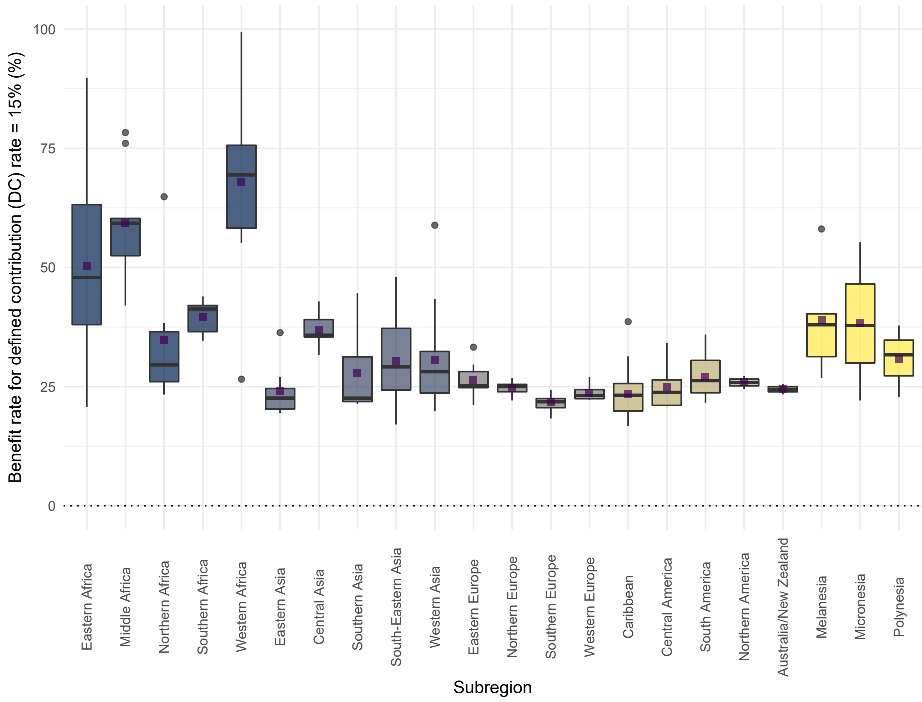
Source: Authors' calculations, based on United Nations (2017b).
Notes: Ratio of per capita benefits to per capita net wages (ϕ) = 100%. Vertical dotted line indicates the median of the distribution.

Figure 5 – Contribution rate (*con*) of DB in 2100 by subregions



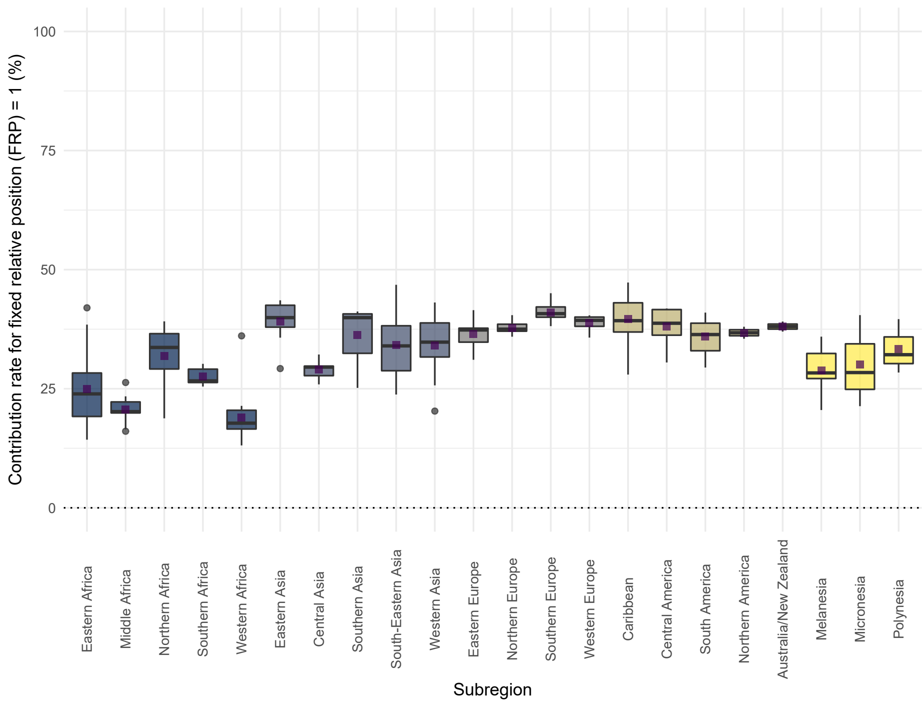
Source: Authors' calculations, based on United Nations (2017b).
Notes: Benefit rate (*ben*) = 100%. Square indicates the mean of the distribution.

Figure 6 – Benefit rate (*ben*) of DC in 2100 by subregions



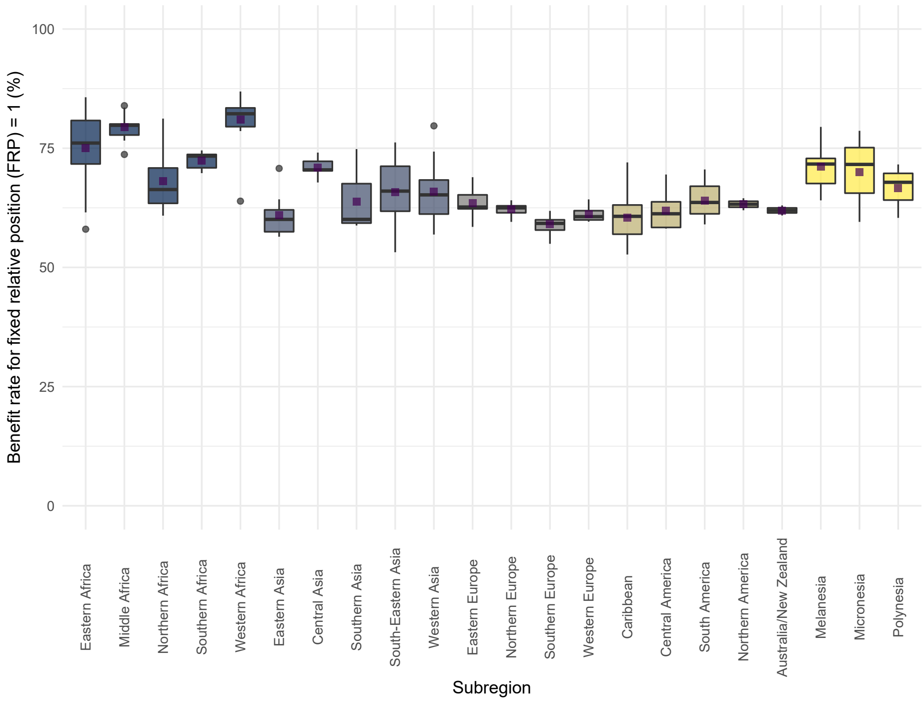
Source: Authors' calculations, based on United Nations (2017b).
Notes: Contribution rate (*con*) = 15%. Square indicates the mean of the distribution.

Figure 7 – Contribution rate (*con*) of FRP in 2100 by subregions



Source: Authors' calculations, based on United Nations (2017b).
Notes: Ratio of per capita benefits to per capita net wages (ϕ) = 100%. Square indicates the mean of the distribution.

Figure 8 – Benefit rate (*ben*) of FRP in 2100 by subregions



Source: Authors' calculations, based on United Nations (2017b).
Notes: Ratio of per capita benefits to per capita net wages (ϕ) = 100%. Square indicates the mean of the distribution.

EQUIVALENT RETIREMENT AGES, PAY-AS-YOU-GO SYSTEMS AND WORLD POPULATION AGING

We assess the equivalent retirement age (ERA) given by the ratio of expected years in retirement to expected years in work (Equation 15), and analyze how much it buffers the burden of population aging on contributors and beneficiaries in PAYG systems. Figure 9 plots the density of equivalent retirement ages (ERA) for selected periods and all regions with a reference vertical line at 65 years; and Figure 10 details its distribution in 2100 by subregions. Figures 11 to 14 present the densities of contribution rates (*con*) and benefit rates (*ben*) of DB, DC and FRP PAYG systems and ERA for selected years and all regions; and Figures 15 to 18 detail their distribution in 2100 by subregions.

As a consequence of increasing ratios of expected years in retirement to expected years in work, equivalent retirement ages (ERA) rise in all regions from 65 years in 1950–1955 to median values around 70 years in 2000–2005, to about 75 years in 2050–2055, and eventually 77.8 years in Africa, 79.3 years in Asia, 78.3 years in Europe, 78.9 years in the Americas, and 79.3 years Oceania. In 2100, the subregions with the highest median ERA are Northern Africa in Africa (81.0 years), Southern Asia and Eastern Asia (82.7 years) in Asia, Southern Europe and Western Europe in Europe (79.4 years), Central America in the Americas (80.8 years), and Polynesia in Oceania (83.9 years).

Much of the debate about adjusting the retirement age based on gains in longevity focus on both levels and changes in the levels of the equivalent retirement age (ERA). But the noteworthy characteristic from our results is both the homogeneity of ERA among subregions that have different rejuvenating effect of deaths, and the heterogeneity of ERA among subregions that have similar rejuvenating effect of deaths. For example, in 2100, homogeneity of ERA and different cumulative rejuvenating effect of deaths are observed for Northern America and Australia/New Zealand compared with Easter Africa and Middle Africa, for Eastern Europe and Northern Europe compared to Southern Africa and Western Africa, and in that twelve of the twenty-two subregions have median ERA within the range 77 years to 80 years.[16] Also in 2100, heterogeneity of ERA and similar cumulative rejuvenating effect of deaths are noted within the four Europe subregions, and between Melanesia, Micronesia and Polynesia.

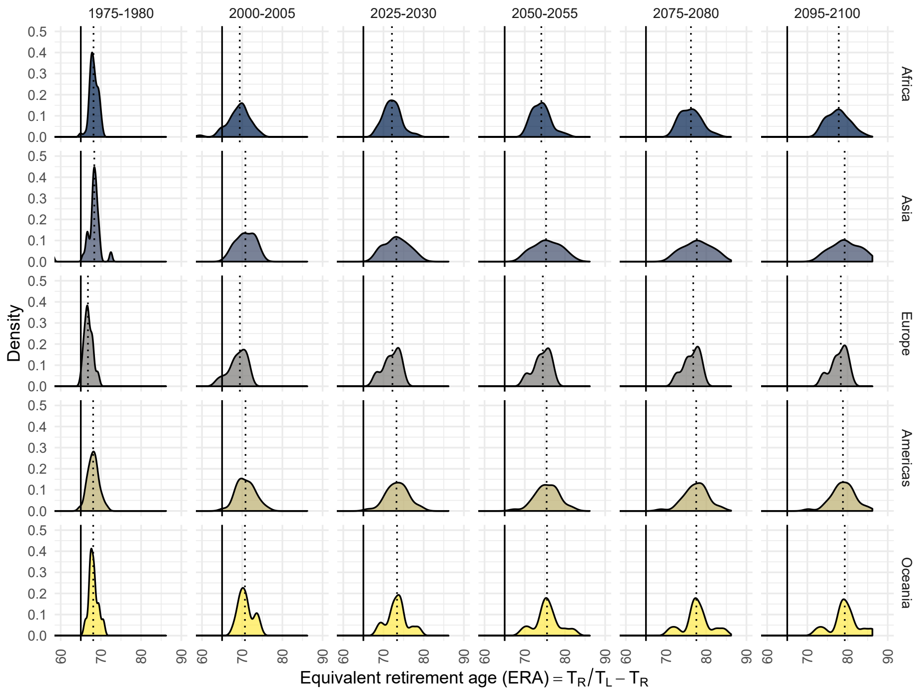
Although equivalent retirement ages (ERA) do buffer the burden of population aging, our results indicate that ERA frequently *over-buffer* this burden when there are mismatches between ERA and the rejuvenating effect of deaths, that is, ERA increase the age of entry into retirement (R) more than necessary, resulting in lower variable contribution rates (*con*) and higher variable benefit rates (*ben*) than in the base year. Actually, between 1950 and 2100, Africa, Asia, the Americas and Oceania observe median variable *con* that first decrease then increase, and median variable *ben* that initially increase then decrease. In defined benefit (DB) PAYG systems, the median *con* in Africa decreases from 7.2% in 1950 to 3.3% in 2025 then increases to 7.8% in 2100, in Asia decreases from 7.8% in 1950 to 4.3% in 2010 then increases to 17.3% in 2100, in the Americas decreases from 8.7% in 1950 to 6.9% in 1990 then increases to 22.1% in 2100, and in Oceania decreases from 6.9% in 1950 to 4.5% in 2010 then increases to 14.4% in 2100. In Europe, the median *con* remains between 13.7% and 17.2% from 1950 to 2020, then increases to 25.7% in 2100. Likewise, in defined contribution (DC) PAYG systems, the median *ben* in Africa increases from 206.9% in 1950 to about 454.0% in 2025 then decreases to 191.9% in 2100, in Asia increases from 193.4% in 1950 to 349.3% in 2010 then decreases to close 86.7% in 2100, in the Americas increases from 172.7% to 219.0% in 1990 then decrease to 68.0% in 2100, and in Oceania increases from 217.7% in 1950 to 332.4% in 2010 then decreases to 104.5% in 2100. In Europe, the median *ben* varies between 109.0% and 87.0% from 1950 to 2020, then decreases to 58.4% in 2100.

The same happens in fixed relative position (FRP) PAYG systems, yet to a lesser degree because FRP PAYG systems distribute the risk of population aging between contributors and beneficiaries, and thus dilute any over-buffering of population aging between *con* and *ben*. In FRP PAYG systems, the median *con* in Africa decreases from 6.6% to in 1950 to 3.2% in 2025 then increases to 7.2% in 2100, in Asia decreases from 7.2% to in 1950 to 4.1% in 2010 then increases to 14.8% in 2100, in the Americas decreases from 8.0% in 1950 to 6.4% in 1990 then increases to 18.1% in 2100, and in

[16] In decreasing order of ERA (median ERA, median cumulative rejuvenating effect of deaths): Western Europe (79.4 years, 58.5 years), Southern Europe (79.4 years, 60.6 years), Melanesia (79.3 years, 29.1 years), South America (79.2 years, 38.8 years), South-Eastern Asia (79.0 years, 36.8 years), Australia/New Zealand (78.9 years, 49.1 years), Western Asia (78.7 years, 27.8 years), the Caribbean (78.2 years, 49.5 years), Eastern Africa (78.1 years, 17.5 years), Northern America (77.9 years, 51.2 years), Middle Africa (77.7 years, 16.8 years), and Northern Europe (77.1 years, 59.5 years) (Fernandes, 2019; Fernandes & Turra, 2019).

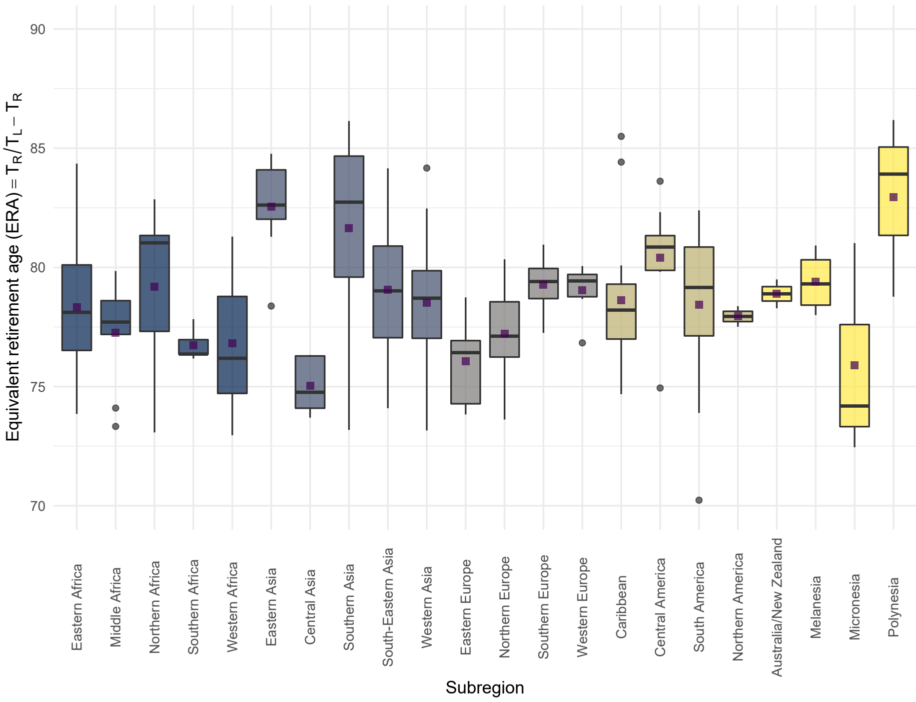
Oceania decreases from 6.4% in 1950 to just below 4.3% in 2010 then increases to 12.6% in 2100. In Europe, the median *con* remains between 12.1% and 14.7% from 1950 to 2020, then increases to 20.4% in 2100. Likewise, still in FRP PAYG systems, the median *ben* in Africa increases from 93.2% in 1950 to 96.8% in 2025 then decreases to 92.8% in 2100, in Asia increases from 92.8% in 1950 to 95.9% in 2010 then decreases to 85.2% in 2100, in the Americas increases from 92% to 93.6% in 1990 then decreases to 81.9% in 2100, and in Oceania increases from 93.6% in 1950 to 95.7% in 2010 then decreases to 87.4% in 2100. In Europe, the median *ben* varies between 87.9% and 85.3% from 1950 to 2020, then decreases to 79.6% in 2100.

Figure 9 – Density of equivalent retirement age (ERA) by selected periods and regions



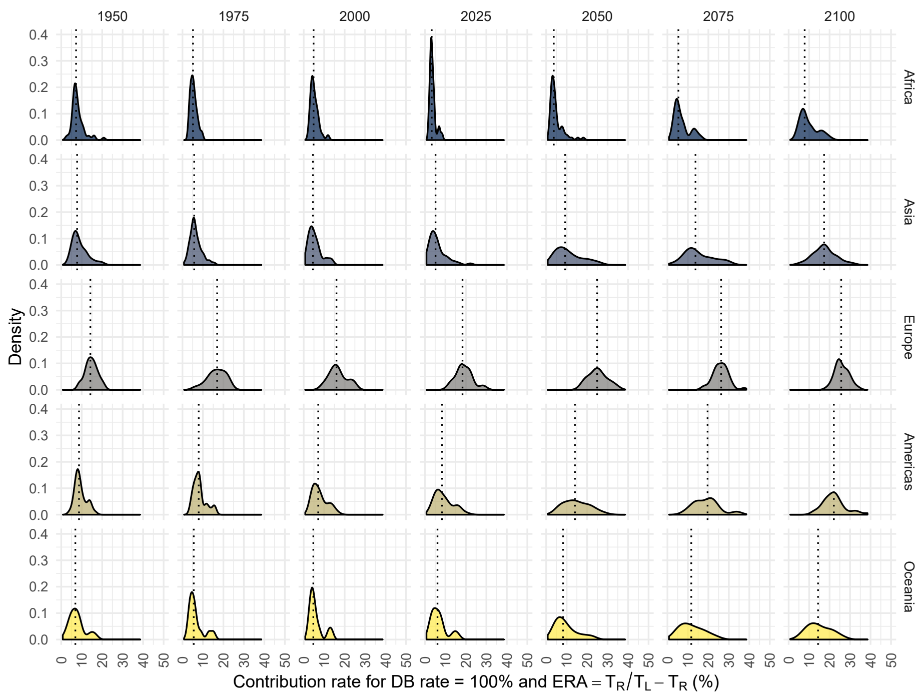
Source: Authors' calculations, based on United Nations (2017b).
Notes: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Vertical dotted line indicates the median of the distribution.

Figure 10 – Equivalent retirement age (ERA) in 2100 by subregions



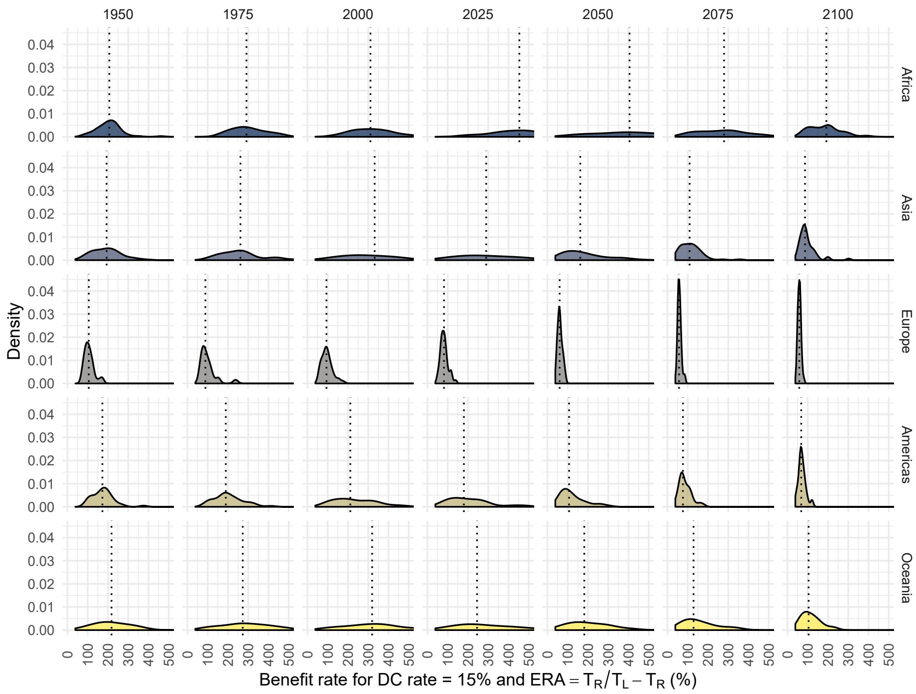
Source: Authors' calculations, based on United Nations (2017b).
Notes: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Square indicates the mean of the distribution.

Figure 11 – Density of the contribution rate (*con*) of DB and ERA by selected periods and regions



Source: Authors' calculations, based on United Nations (2017b).
Notes: Benefit rate (*ben*) = 100%. Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Vertical dotted line indicates the median of the distribution.

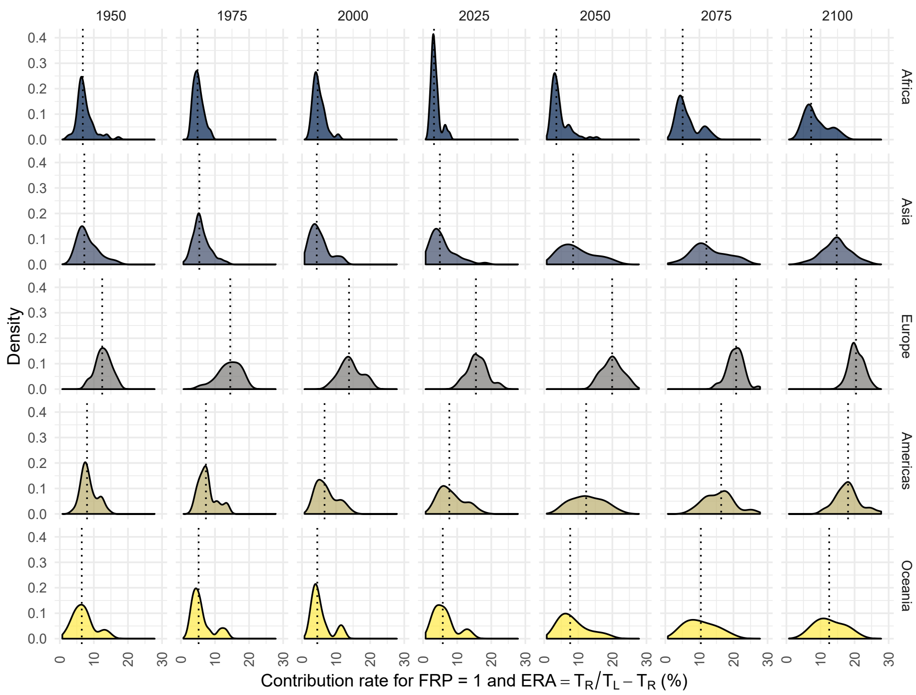
Figure 12 – Density of the benefit rate (*ben*) of DC and ERA by selected periods and regions



Source: Authors' calculations, based on United Nations (2017b).

Notes: Contribution rate (*con*) = 15%. Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Vertical dotted line indicates the median of the distribution.

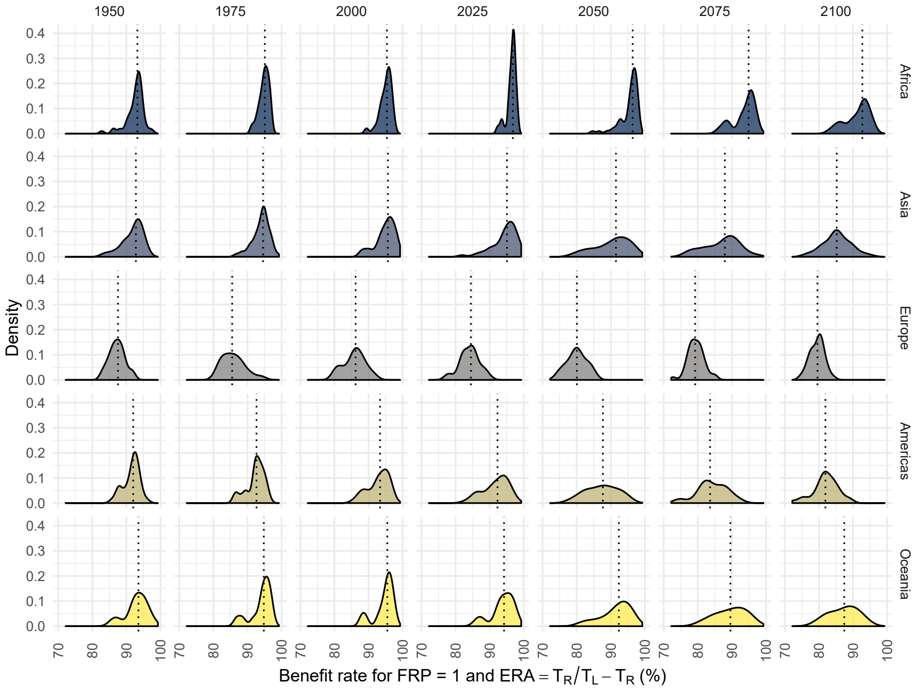
Figure 13 – Density of the contribution rate (*con*) of FRP and ERA by selected periods and regions



Source: Authors' calculations, based on United Nations (2017b).

Notes: Ratio of per capita benefits to per capita net wages (ϕ) = 100%. Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Vertical dotted line indicates the median of the distribution.

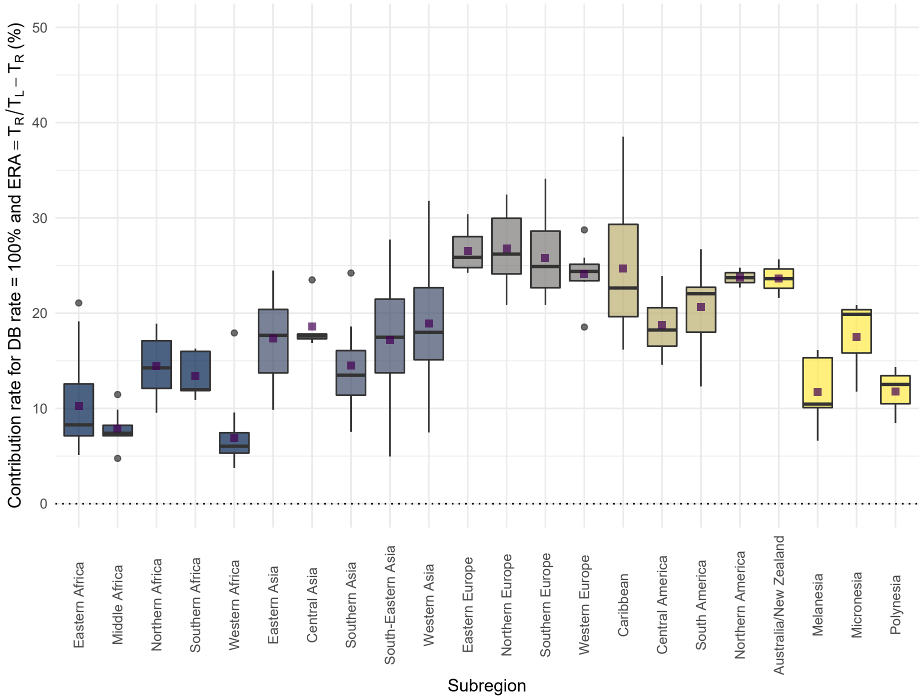
Figure 14 – Density of the benefit rate (*ben*) of FRP and ERA by selected periods and regions



Source: Authors' calculations, based on United Nations (2017b).

Notes: Ratio of per capita benefits to per capita net wages (ϕ) = 100%. Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Vertical dotted line indicates the median of the distribution.

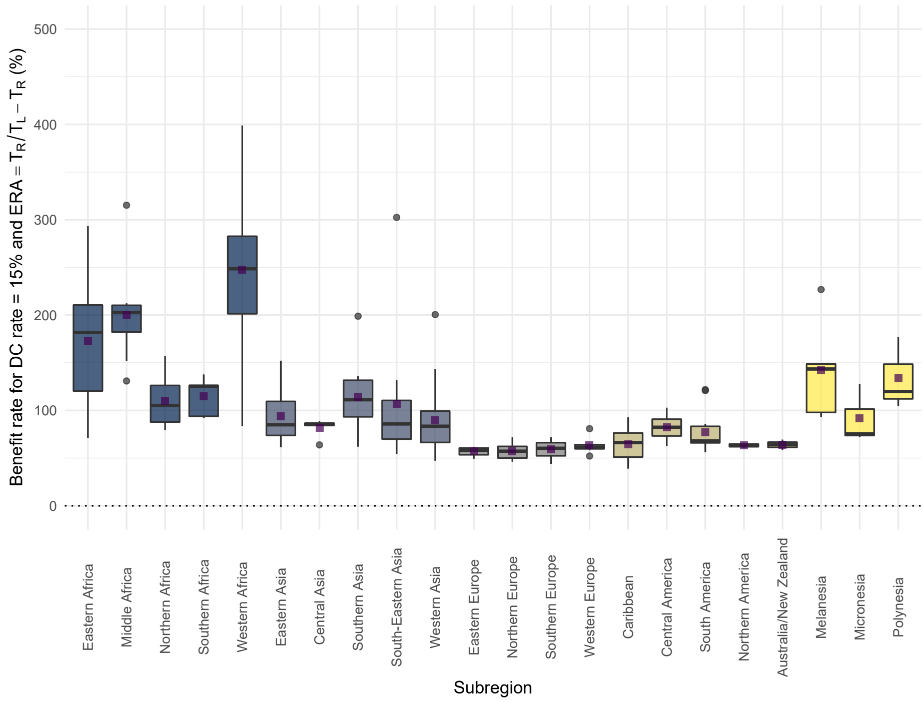
Figure 15 – Contribution rate (*con*) of DB and ERA in 2100 by subregions



Source: Authors' calculations, based on United Nations (2017b).

Notes: Benefit rate (*ben*) = 100%. Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Square indicates the mean of the distribution.

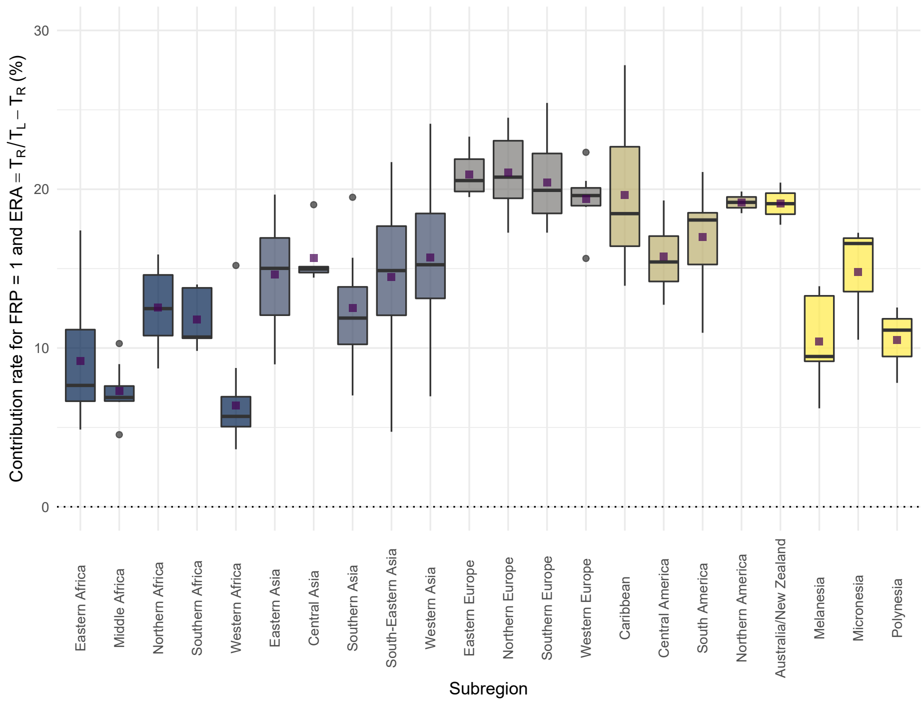
Figure 16 – Benefit rate (*ben*) of DC and ERA in 2100 by subregions



Source: Authors' calculations, based on United Nations (2017b).

Notes: Contribution rate (*con*) = 15%. Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Square indicates the mean of the distribution.

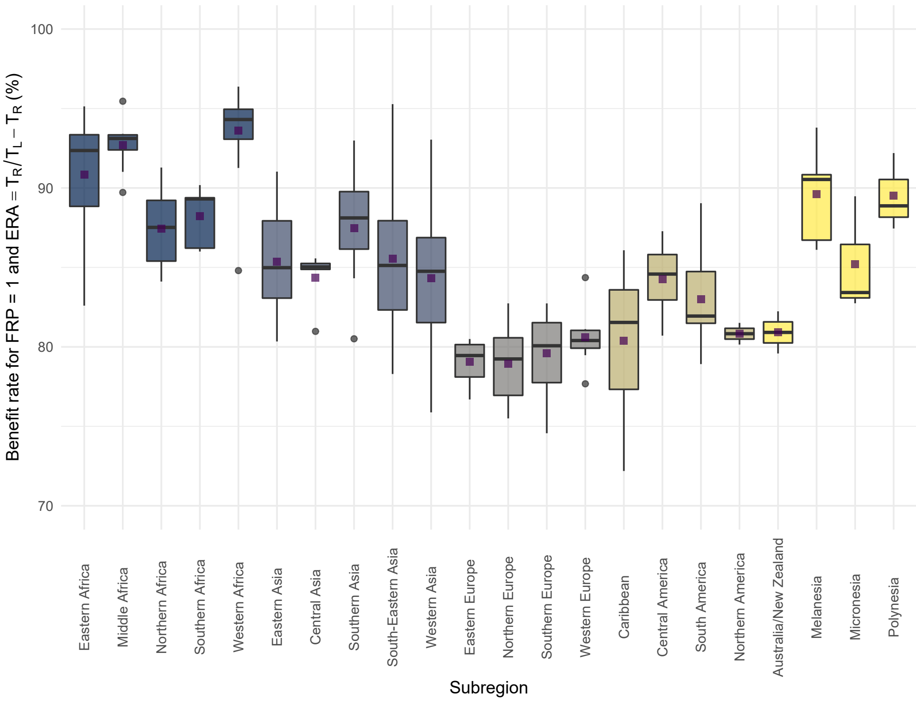
Figure 17 – Contribution rate (*con*) of FRP and ERA in 2100 by subregions



Source: Authors' calculations, based on United Nations (2017b).

Notes: Ratio of per capita benefits to per capita net wages (ϕ) = 100%. Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Square indicates the mean of the distribution.

Figure 18 – Benefit rate (*ben*) of FRP and ERA in 2100 by subregions



Source: Authors' calculations, based on United Nations (2017b).

Notes: Ratio of per capita benefits to per capita net wages (ϕ) = 100%. Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Square indicates the mean of the distribution.

EQUIVALENT RETIREMENT AGES EFFECTIVENESS

Equivalent retirement ages (ERA) ineffectively over-buffer population aging because PAYG systems are established on population characteristics, while ERA are based on life cycle characteristics. Fundamentally, population characteristics change because mortality, fertility or migration change, while life cycle characteristics change because mortality or migration change (Preston, 1982; Preston & Coale, 1982).[17] Thus, equivalent retirement ages (ERA) may not effectively buffer the impact of population aging if the contribution of mortality to changes in the population age distribution is only moderate.[18] Therefore, PAYG systems should contemplate the role of the rejuvenating effect of deaths and, by extension, the stages of the demographic transition, in the definition of retirement age policies.

[17] Preston (1982) formulated measures for both the prevalence of an attribute G in a population at a moment in time (G_P) and the prevalence of an attribute G over the course of the life cycle (G_L) according to stable populations that are closed to migration. Nevertheless, the formulas of G_P and G_L may be extended to accommodate non-stable populations and migration. Essentially, the effect of migration on a cohort's or a population's size is analogous to the effect of mortality (Preston & Coale, 1982).

[18] Basically, old age dependency ratios (OADR) are Preston (1982)'s G_P, while equivalent retirement ages (ERA) are Preston (1982)'s G_L. Rigorously, old age dependency ratios (OADR) are the ratio between two G_P's (G_P^{20-64}/G_P^{65+}), and ERA measured by the ratio of expected years in retirement to expected years in work are the ratio between two G_L's (G_L^{20-64}/G_L^{65+}). In the one (G_P^{20-64} or G_L^{20-64}) the characteristic G is to be 20 to 64 years of age, in the other (G_P^{65+} or G_P^{65+}) the characteristic G is to be 65 years of age and older.

We may measure the level of population aging relative to the base year 1950 by calculating the ratio of the old age dependency ratio (OADR) observed at year t ($OADR(t)$) to the OADR observed in 1950 ($OADR(1950)$),

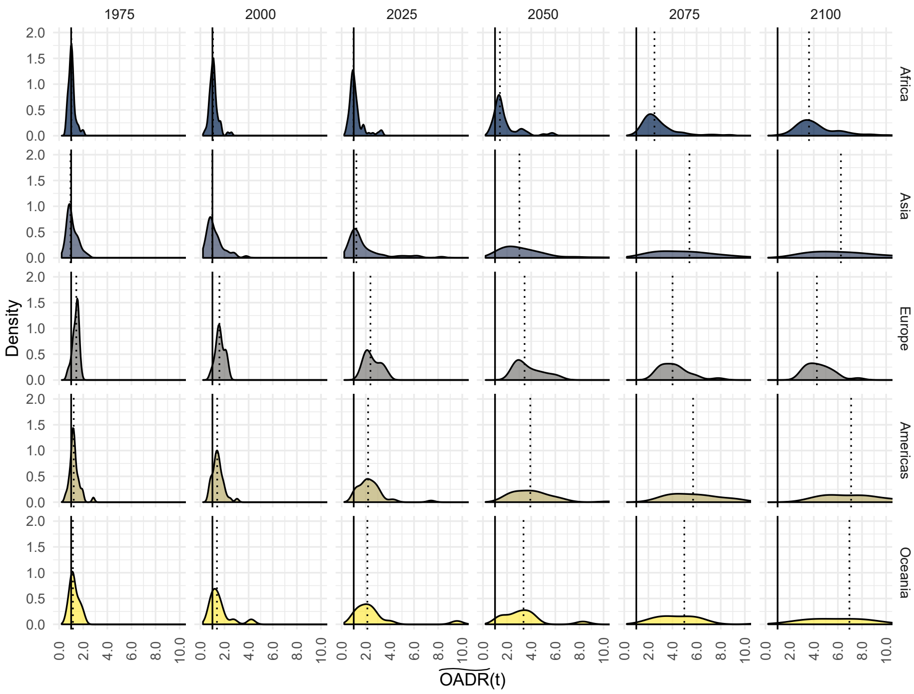
$$\widetilde{OADR}(t) = \frac{OADR(t)}{OADR(1950)} \tag{16}$$

Likewise, we may assess the effectiveness of equivalent retirement ages (ERA) relative to the base year 1950 by calculating the ratio of the equivalent old age dependency ratio (EOADR) observed at year t ($EOADR(t)$) to the EOADR observed in 1950 ($EOADR(1950)$),

$$\widetilde{EOADR}(t) = \frac{EOADR(t)}{EOADR(1950)} \tag{17}$$

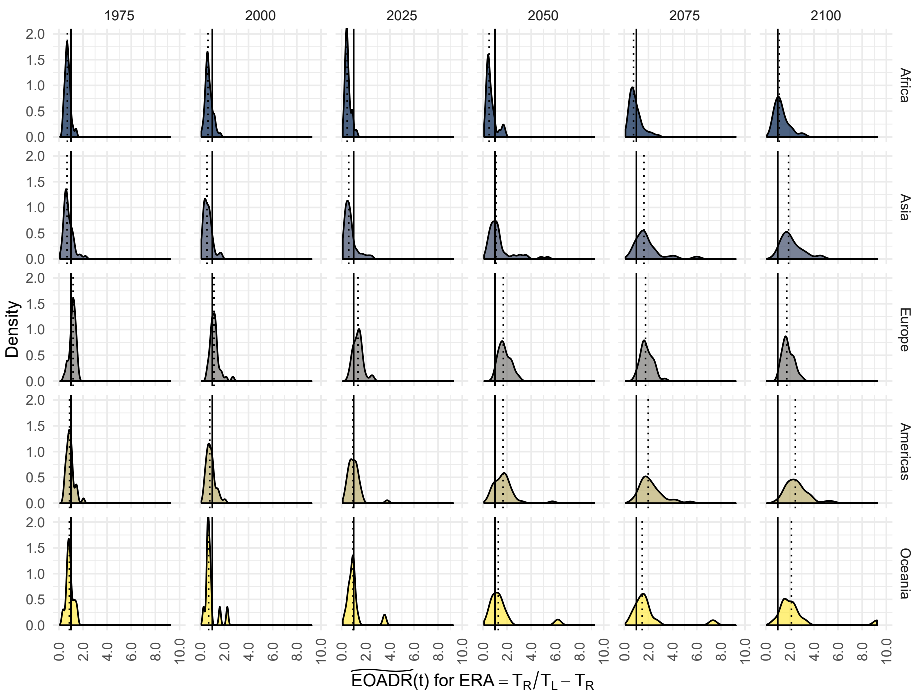
Figures 19 and 20 respectively present the density of $\widetilde{OADR}(t)$ and $\widetilde{EOADR}(t)$ for selected years and all regions with a reference vertical line at 1. Eventually in 2100, the median of $\widetilde{OADR}(t)$ is 3.6 in Africa, 6.2 in Asia, 4.3 in Europe, 7.1 in the Americas, and 7.0 in Oceania. As we could anticipate, $\widetilde{EOADR}(t)$ are quite lower than $\widetilde{OADR}(t)$; in 2100, its median is 1.04 in Africa, 1.9 in Asia, 1.7 in Europe, 2.5 in the Americas, and 2.1 in Oceania.

Figure 19 – Density of $\widetilde{OADR}(t)$ by selected periods and regions



Source: Authors' calculations, based on United Nations (2017b).
Note: Vertical dotted line indicates the median of the distribution.

Figure 20 – Density of $\widetilde{EOADR}(t)$ by selected periods and regions



Source: Authors' calculations, based on United Nations (2017b).
Notes: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Vertical dotted line indicates the median of the distribution.

Fundamentally, if $\widetilde{EOADR}(t) = 1$ then equivalent retirement ages (ERA) sustain in year t the same equivalent old age dependency ratios (EOADR) that were observed in the base year 1950, that is, they *effectively* compensate changes in populations age distributions relatively to 1950. But a proper analysis of ERA effectiveness should compare $\widetilde{EOADR}(t)$ with $\widetilde{OADR}(t)$ into *effectiveness categories* as we depict in Figure 21. Populations that are in the line BEH are neither aging nor rejuvenating, those that are above this line are aging, and those that are below this line are rejuvenating, always relatively to ERA's base year. Populations that are in the *compensation limit line*[19] (line AEI) have ERA equal to R, ultimately there is no compensation whatsoever to any changes in populations age distributions relatively to ERA's base year. If populations are aging, those that are above the compensation limit line[20] (triangle EHI) observe ERA which are less than R and thus *overload* the burden of population aging; populations that are below this line[21] observe ERA which are higher than R and thus either *under-compensate*, that is, increase R less than necessary (triangle EFI), or *effectively compensate*, namely, increase R as much as necessary, (line EF), or *over-compensate*, specifically, increase R more than necessary (square BEFC). Similarly, if populations are rejuvenating and R do not need to change, those that are below the compensation limit line (triangle ABE) observe ERA which are higher than R and thus *over-rejuvenate* populations; populations that are above this line observe ERA which are lower than R and thus either *under-neutralize*, that is, decrease R less than necessary to increase EOADR (triangle ADE), or *effectively neutralize*, specifically, decrease R as much as necessary to sustain the same EOADR (line DE), or *over-neutralize*, namely, decrease R more than necessary to increase EOADR (square DGHE).

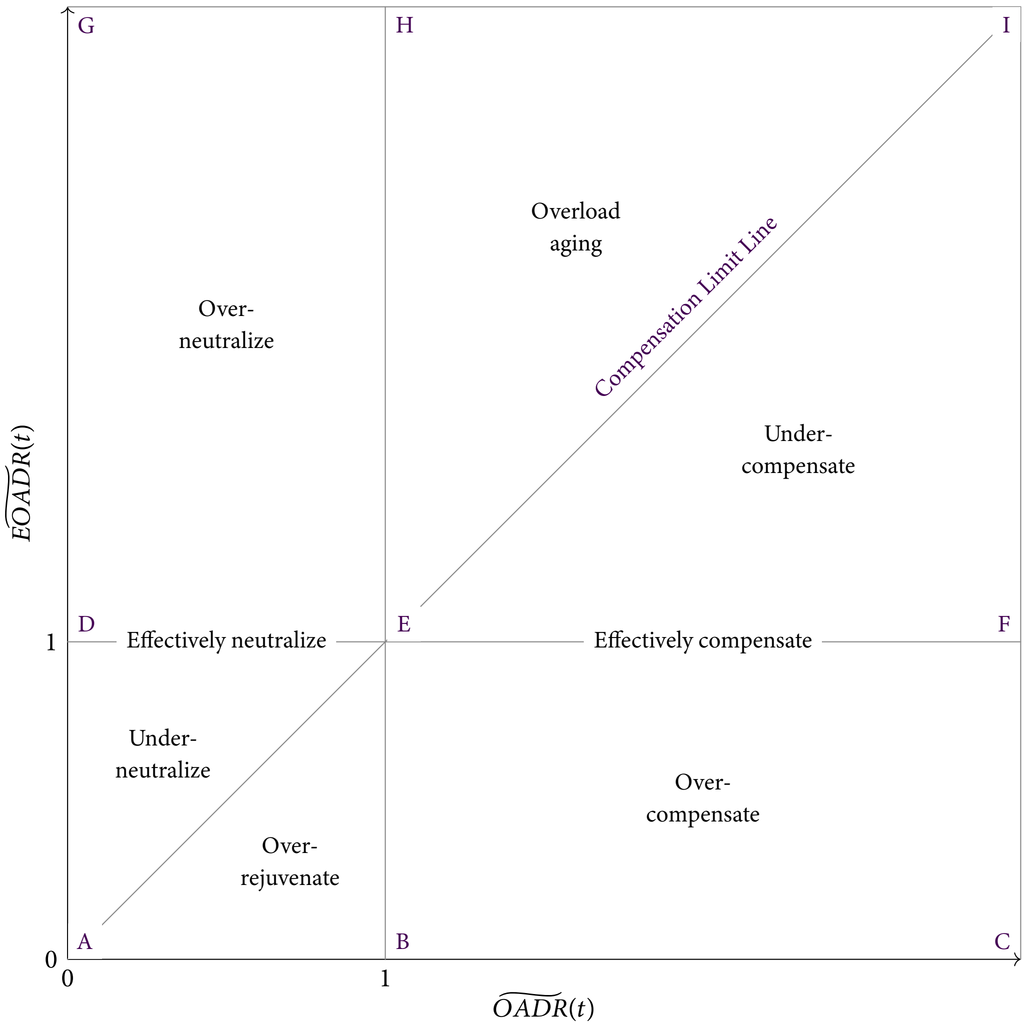
We plot $\widetilde{OADR}(t)$ by $\widetilde{EOADR}(t)$ in Figure 22 and detail it by subregions in Figure 23. Most populations under-compensate, over-compensate, or over-rejuvenate changes in the OADR, with very few discernible populations in the other effectiveness categories. Over-compensate and over-rejuvenate populations are more evident in Africa, Asia, Southern Europe, the Caribbean, South America and Melanesia, and are mostly associated with lower rejuvenating effects of deaths. Populations in Eastern Europe and Northern Europe markedly under-compensate and are linked to higher rejuvenating effects of deaths. Northern America and Australia/New Zealand are closer to effectively compensate.

[19] $\widetilde{EOADR}(t) = \widetilde{OADR}(t) \implies EOADR(t) = OADR(t)$.

[20] $\widetilde{EOADR}(t) > \widetilde{OADR}(t) \implies EOADR(t) > OADR(t)$.

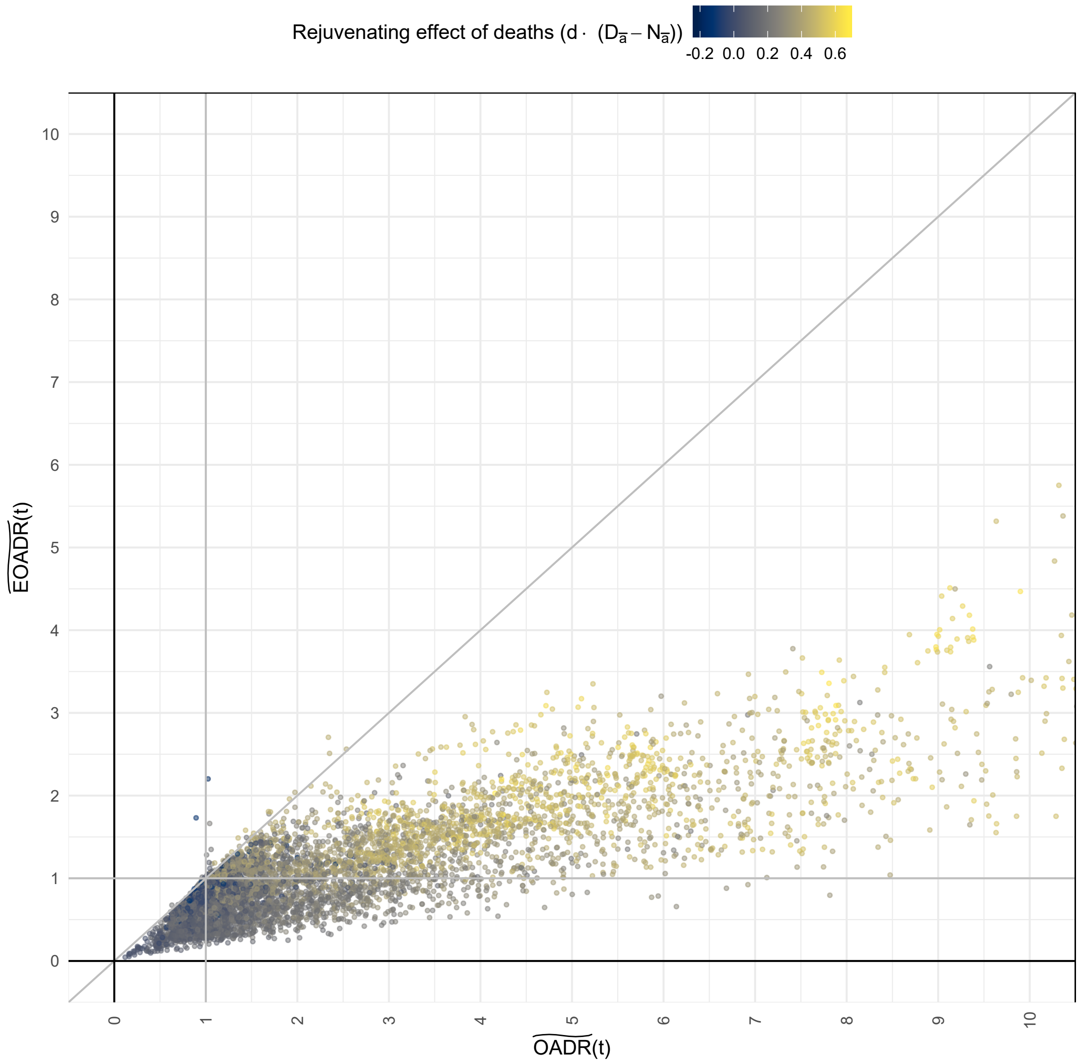
[21] $\widetilde{EOADR}(t) < \widetilde{OADR}(t) \implies EOADR(t) < OADR(t)$.

Figure 21 – Equivalent retirement age (ERA) effectiveness categories measured via $\widetilde{OADR}(t)$ by $\widetilde{EOADR}(t)$



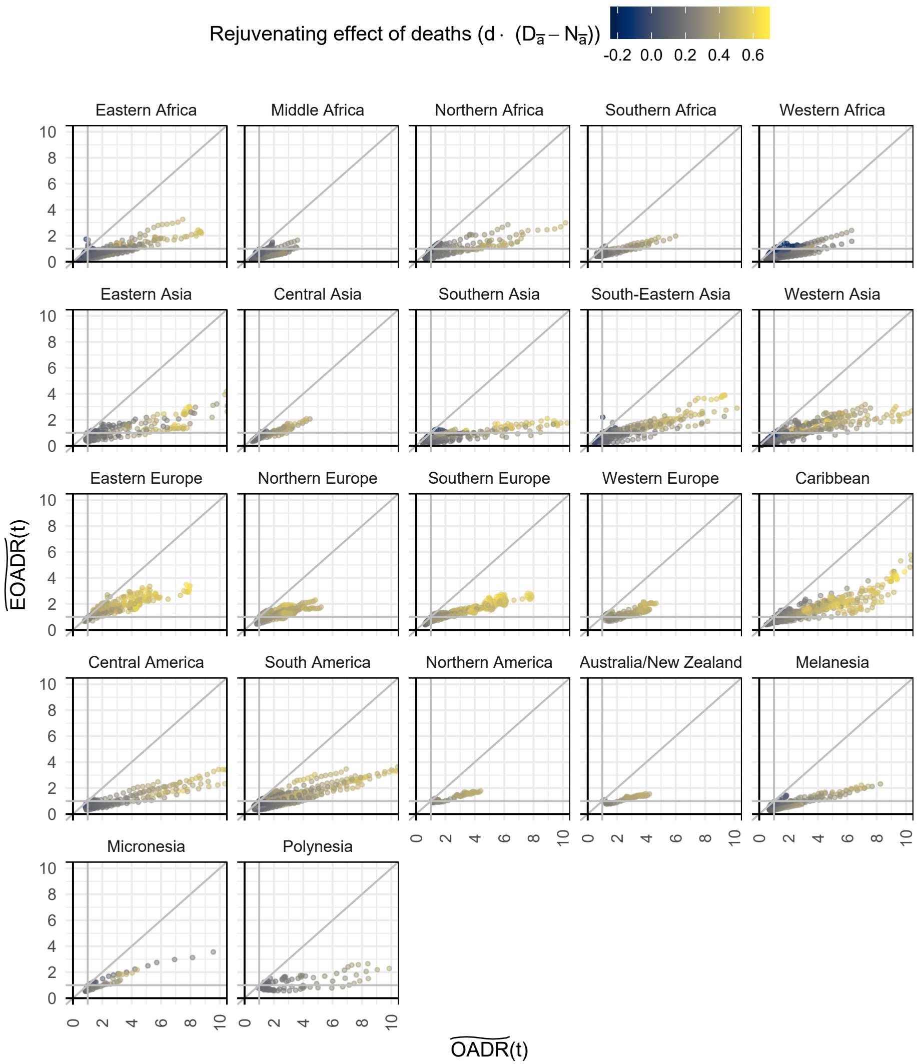
Source: Authors' creation.

Figure 22 – $\widetilde{OADR}(t)$ by $\widetilde{EOADR}(t)$



Source: Authors' calculations, based on United Nations (2017b).
Note: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$.

Figure 23 – $\widetilde{OADR}(t)$ by $\widetilde{EOADR}(t)$ and subregions



Source: Authors' calculations, based on United Nations (2017b).
Note: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$.

We group the number of countries by ERA effectiveness category, and rejuvenating effect of deaths interval and respective stage of the demographic transition in Table 6.[22] For a total of 6,030 populations, ERA over-rejuvenate 19% (1,173), over-compensate 30% (1,794), effectively compensate 6% (353), and under-compensate 44% (2,634), with 1% (76) in the remaining categories. For 391 populations that have rejuvenating effects of deaths between -0.2 and 0.0 (i.e., stages 1 to 1A of the demographic transition), ERA over-rejuvenate 57% (223) and over-compensate 28% (109). For 2,251 populations that have rejuvenating effects of deaths between 0.0 and 0.2 (i.e., stages 2 to 3), ERA over-rejuvenate 39% (1,091) and over-compensate 46% (1,151). For 1,925 populations that have rejuvenating effects of deaths between 0.2 and 0.4 (i.e., first half of stage 4), ERA over-compensate 32% (625), effectively compensate, 10% (194), and under-compensate 52% (999). For 1,383 populations that have rejuvenating effects of deaths between 0.4 and 0.6 (i.e., second half of stage 4), ERA under-compensate 96% (1,326). Particularly, for the 353 populations that ERA effectively compensate, 55% (194) are in the first half of stage 4. Thus, if the OADR is aging relatively to ERA's base year, on the one hand, the lower the rejuvenating effect of deaths, the higher the probability that ERA over-compensate; on the other hand, the higher the rejuvenating effect of deaths, the higher the likelihood that ERA effectively compensate or under-compensate.

[22] In Table 6, the stage of the demographic transition is determined exclusively by rejuvenating effect of deaths (Fernandes, 2019; Fernandes & Turra, 2019). Also, to prevent classifying populations with minimum changes both in the OADR and the EOADR relatively to 1950 in any of the effectiveness categories, we classify populations that have 0.95 =< $\widetilde{OADR}(t)$ <= 1.05 and 0.95 =< $\widetilde{EOADR}(t)$ <= 1.05 as neutral-aging. Also, to avert zero counting of populations in the effectiveness categories effectively neutralize and effectively compensate, albeit populations close to be classified as so, we classify populations that observe $\widetilde{OADR}(t)$ > 1.05 and 0.95 =< $\widetilde{EOADR}(t)$ <= 1.05 as effectively compensate; and populations that observe $\widetilde{OADR}(t)$ < 0.95 and 0.95 =< $\widetilde{EOADR}(t)$ <= 1.05 as effectively neutralize. We adjust the other ERA effectiveness categories accordingly.

Table 6 – Number of countries by equivalent retirement age (ERA) effectiveness category, and rejuvenating effect of deaths interval and respective stage of the demographic transition

Equivalent retirement age (ERA) effectiveness category	Rejuvenating effect of deaths								Total	Example Countries
	−0.2, 0.0	0.0	0.0, 0.2	0.2	0.2, 0.4	0.4, 0.6	0.6	> 0.6		
	Stage of the demographic transition									
	1	1A	2	3	4	4	4A	5		
Neutral-aging	2	0	20	2	17	1	0	0	42	Estonia (1990–1995) New Zealand (1970–1975) Zambia (2000–2005)
Over-neutralize	1	0	0	0	0	0	0	0	1	Rwanda (1990–1995)
Effectively neutralize	0	0	0	0	0	0	0	0	0	…
Under-neutralize	0	0	2	1	0	0	0	0	3	South Africa (1980–1985) Guyana (1990–1995) Botswana (2000–2005)
Over-rejuvenate	139	84	786	82	82	0	0	0	1,173	Uruguay (1960–1965) China (1970–1975) Philippines (2010–2015)
Over-compensate	49	60	751	291	625	18	0	0	1,794	Mexico (1995–2000) United Kingdom (2015–2020) India (2030–2035)
Effectively compensate	18	10	74	30	194	27	0	0	353	United States (2020–2025) Chile (2025–2030) France (2030–2035)
Under-compensate	19	8	135	67	999	1,326	62	18	2,634	Switzerland (1985–1990) Italy (2020–2025) Brazil (2035–2040)
Overload aging	1	0	5	5	8	11	0	0	30	Norway (1965–1970) Russian Federation (2000–2005) Zimbabwe (2005–2010)
Total	229	162	1,773	478	1,925	1,383	62	18	6,030	

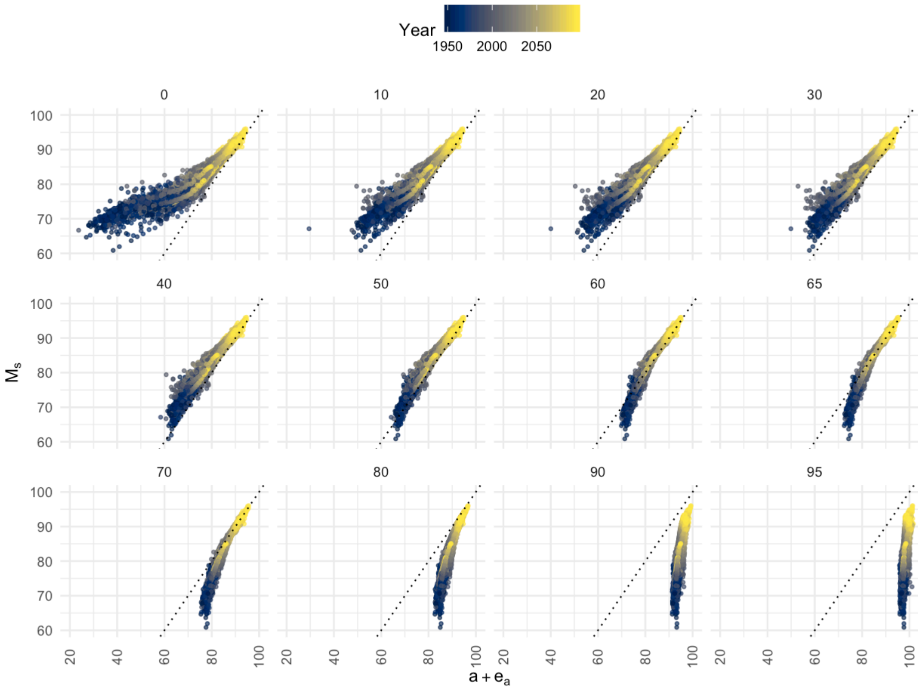
Source: Authors' creation and calculations, based on United Nations (2017b).
Notes: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Neutral-aging = 0.95 =< $\widetilde{OADR}(t)$ <= 1.05 and 0.95 =< $\widetilde{EOADR}(t)$ <= 1.05; Effectively compensate = $\widetilde{OADR}(t)$ > 1.05 and 0.95 =< $\widetilde{EOADR}(t)$ <= 1.05; Effectively neutralize = $\widetilde{OADR}(t)$ < 0.95 and 0.95 =< $\widetilde{EOADR}(t)$ <= 1.05. The other ERA effectiveness categories are adjusted accordingly. Stage of the demographic transition determined exclusively by rejuvenating effect of deaths $(d(t) \cdot [D_{\overline{a}}(t) - N_{\overline{a}}(t)])$.
…: Not applicable.

EQUIVALENT RETIREMENT AGES BASED ON THE MODAL AGE AT DEATH

The equivalent ages presented earlier in this paper reflect indicators of longevity based on life expectancy at age a ($\mathring{e}_a$), that is, the mean length of life or the mean age at death at age a. But the modal age at death (M) is a recommended alternative indicator of longevity, because it is only determined by old-age mortality, it is free from bias caused by arbitrary selections of age limits for old-age, and it is also a valuable public health policy guide (e.g., demand for health infrastructures and services are accentuated at ages around M) (Horiuchi et al., 2013; Kannisto, 2001; Missov, Lenart, Nemeth, Canudas-Romo, & Vaupel, 2015).

From birth up to middle-ages, the modal age at death (M) is older than total life expectancy at age a ($a + \mathring{e}_a$) due to the general bimodal and left-skewed age distribution of deaths. At birth, this difference may vary from 30 years in high mortality contexts, to 5 years in low mortality conditions (Canudas-Romo, 2010; Horiuchi et al., 2013). This variance is the result of the combination of two factors: first, while M is determined by old-age mortality only, $\mathring{e}_a$ is dictated by mortality at all ages equal to and above a; second, the general pattern of mortality decline, with its initial steep decline in infant and child mortality, followed by a reduction in young-age and middle-age adult mortality, ultimately followed by a decline in old-age mortality (Horiuchi, 1999; Wilmoth, 2000). Accordingly, from birth up to middle-ages, $\mathring{e}_a$ at first rapidly rises at birth and young ages, and next improves at middle-ages, alongside negligible gains in M. Then, as declines in mortality shift to older ages, improvements in $\mathring{e}_a$ decelerates and M increases. Consequently, from birth up to middle-ages, the difference between M and $a + \mathring{e}_a$ first observes a strong decline, and later stabilizes (Horiuchi et al., 2013). At ages older than middle-ages, however, the patterns and trends between M and $a + \mathring{e}_a$ are quite different. Specifically, when old-age mortality is high, M is nearly equal to $a + \mathring{e}_a$ at 60 years of age, and approximately 5 years younger than $a + \mathring{e}_a$ at 75 years of age. As old-age mortality declines, not only M increases faster than $\mathring{e}_a$, but also the higher the age the slower the increase in $\mathring{e}_a$ (e.g., $\mathring{e}_{75}$ increases slower than $\mathring{e}_{65}$), which ultimately results in M higher than $a + \mathring{e}_a$ (Horiuchi et al., 2013). Our estimates of the modal age at death from senescent mortality (M_s) (Fernandes, 2019) are consistent to the patterns and trends above, as in Figure 24 that plots total life expectancy at age a ($a + \mathring{e}_a$) by M_s and selected ages.

Figure 24 – Total life expectancy at age a ($a + \mathring{e}_a$) by modal age at death from senescent mortality (M_s) and selected ages



Source: Authors' calculations, based on United Nations (2017b).

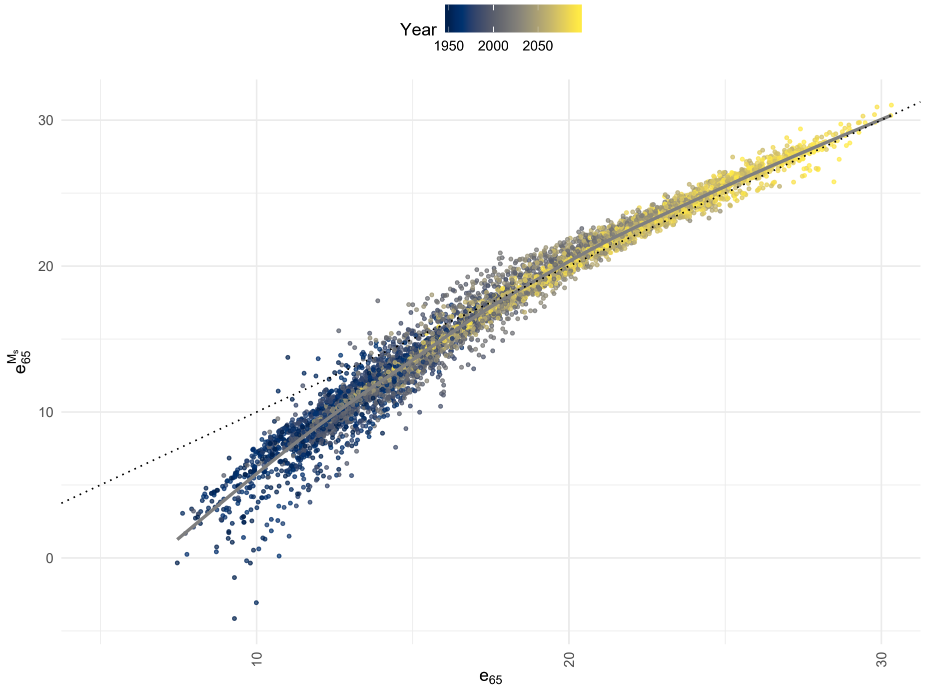
Comparing the total life expectancy at age a ($a + \mathring{e}_a$) with the modal age at death from senescent mortality (M_s) is equivalent to corresponding life expectancy at age a ($\mathring{e}_a$) to $M_s - a$. Consequently, let the difference between M_s and age a be the *modal life expectancy* at age a,

$$\mathring{e}_a^{M_s} = M_s - a \tag{18}$$

As an illustration, for age 65, the modal life expectancy at age a ($\mathring{e}_a^{M_s}$) changes from values that are initially 5 to 10 years younger than $\mathring{e}_a$ to values that are around 1 to 2 years older than life expectancy at age a ($\mathring{e}_a$), as we observe in Figures 25 and 26.[23]

[23] Figure 33 in Appendix — Equivalent retirement ages based on the modal age at death details Figure 26 by subregions.

Figure 25 – Modal life expectancy at age a ($\mathring{e}_a^{M_s}$) by life expectancy at age a ($\mathring{e}_a$) for age 65



Source: Authors' calculations, based on United Nations (2017b).

Figure 26 – Life expectancy at age a ($\mathring{e}_a$) by modal life expectancy at age a minus life expectancy at age a ($\mathring{e}_a^{M_s} - \mathring{e}_a$) for age 65

Source: Authors' calculations, based on United Nations (2017b).

Thus, when old-age mortality declines, adjusting the age of entry into retirement (R) based on gains in the life expectancy at age a ($\mathring{e}_a$) may be less effective than based on gains in the modal life expectancy at age a ($\mathring{e}_a^{M_s}$), because increases in the age of entry into retirement (R) will be slower than the gains in longevity for *most beneficiaries* of the PAYG system. Accordingly, we propose equivalent retirement age (ERA) measures based on the modal life expectancy at age a ($\mathring{e}_a^{M_s}$). Table 7 presents these measures in terms of the modal number of person-years lived above age a ($T_a^{M_s}$) and the number of survivors to age a (l_a).[24,25]

Table 7 – Equivalent retirement age (ERA) by point of measurement and characteristic of measurement based on $\mathring{e}_a^{M_s}$ and in terms of $T_a^{M_s}$ and l_a

Point of measurement	Characteristic of measurement			
	Modal expected years in retirement		Ratio of modal expected years in retirement to modal expected years in work	
Entry into retirement (R)	$\frac{T_R^{M_s}}{l_R}$	(19)	$\frac{T_R^{M_s}}{l_R \cdot (R - L)}$	(20)
Entry into labor force (L)	$\frac{T_R^{M_s}}{l_L}$	(21)	$\frac{T_R^{M_s}}{T_L^{M_s} - T_R^{M_s}}$	(22)

Source: Authors' creation, based on table in Bayo and Faber (1981, p. 4).

We evaluate the equivalent retirement age (ERA) based on $\mathring{e}_a^{M_s}$ given by the ratio of expected modal years in retirement to expected modal years in work measured at the age of entry into the labor force (L) (Equation 22). Figure 27 plots the density of ERA based on $\mathring{e}_a^{M_s}$ for selected periods and all regions with a reference vertical line at 65 years; and Figure 28 details its distribution in 2100 by subregions. Figure 29 presents the density of the difference between ERA based on $\mathring{e}_a^{M_s}$ and ERA based on $\mathring{e}_a$ for selected periods and all regions with a vertical reference line at zero; and Figure 30 plots its distribution in 2100 by subregions.

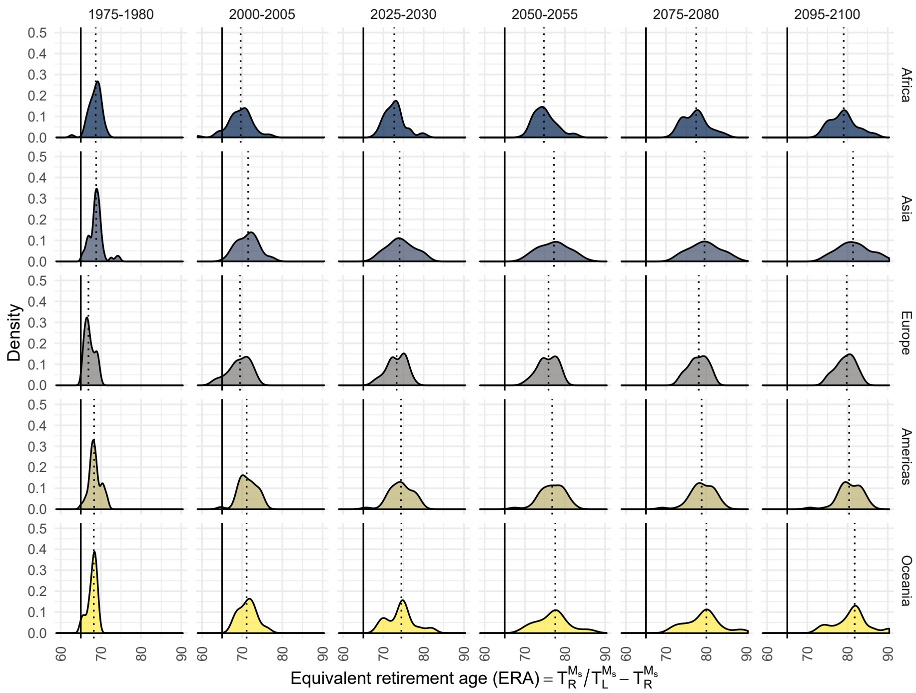
In 2100, the median of the difference between ERA based on $\mathring{e}_a^{M_s}$ and ERA based on $\mathring{e}_a$ is 1.1 years in Africa, 2.2 years in Asia, 1.3 years in Europe, and 1.6 years in the Americas and Oceania. In 2100, the lowest median of the differences are in Western Africa (0.4 year) and Micronesia (0.9 year), and the highest are in Polynesia (3.9 years), Central Asia (3.0 years), and Eastern Asia (2.8 years). Still in 2100, extreme high differences are observed in Cambodia (5.4 years, 87.1 vs 81.7), China (4.8 years, 89.3 vs 84.5), and Mauritius (4.8, 86.8 vs 82.0). Negative differences are present in eight countries, with only two relevant, Honduras (−1.4 years, 79.6 vs 81.0) and Papua New Guinea (−0.7 year, 77.3 vs 78.0).[26]

24 $T_a^{M_s} = \mathring{e}_a^{M_s} \cdot l_a$.

25 Table 9 in Appendix — Equivalent retirement ages based on the modal age at death presents the same measures, in terms of the modal life expectancy at age a ($\mathring{e}_a^{M_s}$) and the number of survivors to age a (l_a).

26 Honduras (−1.37 years), Papua New Guinea (−0.71 years), Mali (−0.31 year), Martinique (−0.23 year), Afghanistan (−0.16 year), Gambia (−0.13 year), Montenegro (−0.046 year), and Comoros (−0.033 year).

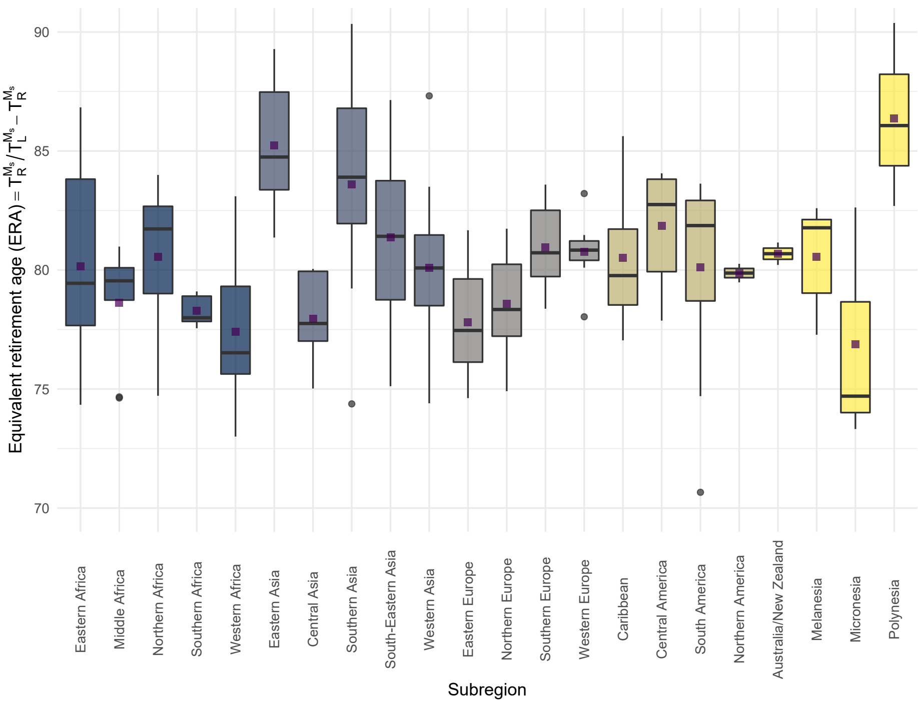
Figure 27 – Density of ERA based on $\mathring{e}_a^{M_s}$ by selected periods and regions



Source: Authors' calculations, based on United Nations (2017b).

Notes: Equivalent retirement age (ERA) = $(T_R^{M_s})/(T_L^{M_s} - T_R^{M_s})$. Vertical dotted line indicates the median of the distribution.

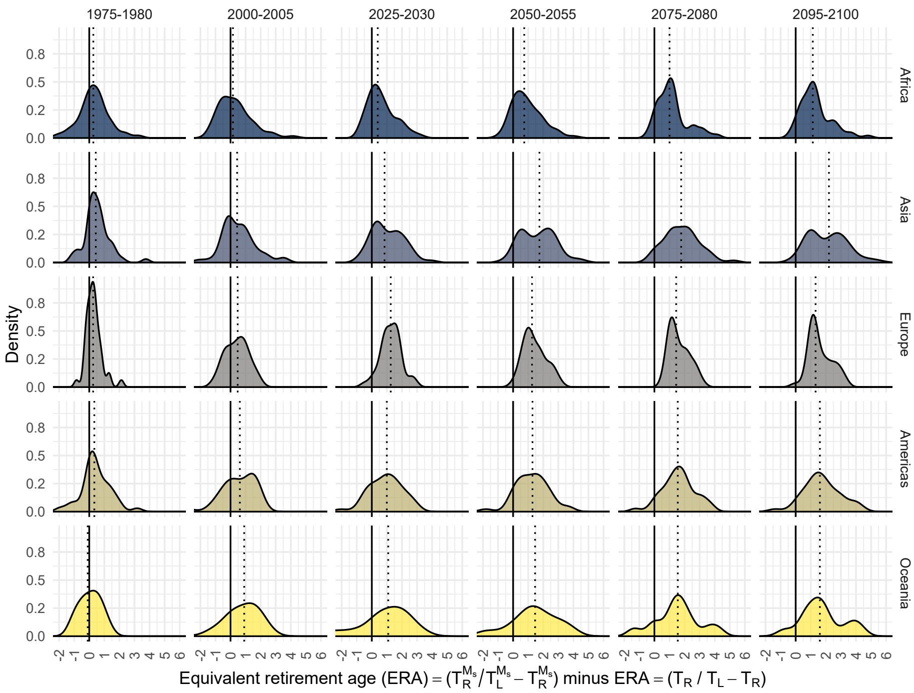
Figure 28 – Equivalent retirement age (ERA) based on $\mathring{e}_a^{M_s}$ in 2100 by subregions



Source: Authors' calculations, based on United Nations (2017b).

Notes: Equivalent retirement age (ERA) = $(T_R^{M_s})/(T_L^{M_s} - T_R^{M_s})$. Square indicates the mean of the distribution.

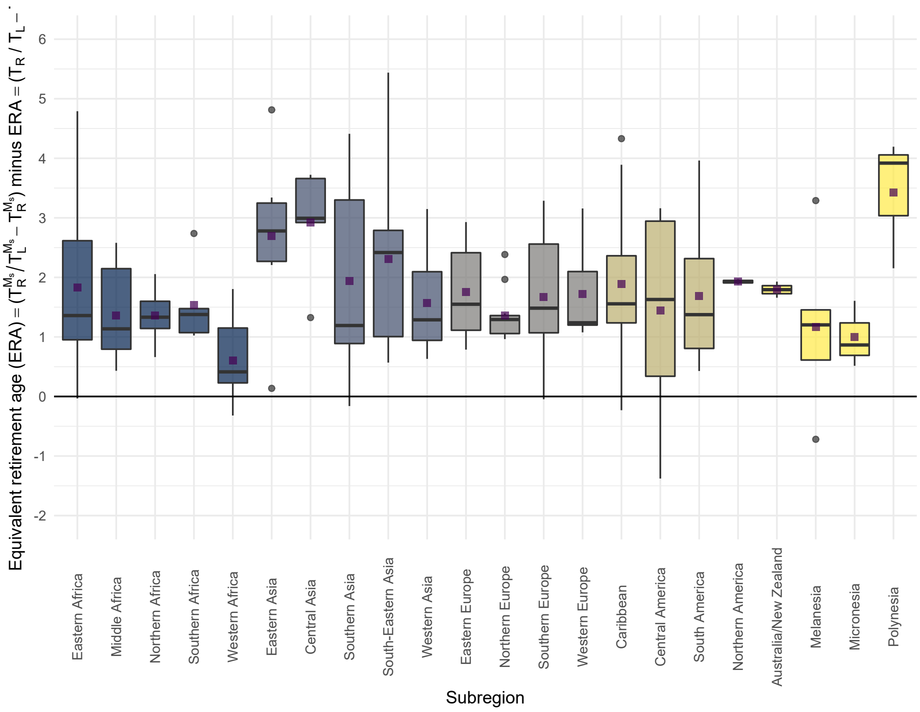
Figure 29 – Density of difference between ERA based on $\mathring{e}_a^{M_s}$ and ERA selected periods and regions



Source: Authors' calculations, based on United Nations (2017b).

Notes: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Equivalent retirement age (ERA) based on $\mathring{e}_a^{M_s} = (T_R^{M_s})/(T_L^{M_s} - T_R^{M_s})$. Vertical dotted line indicates the median of the distribution.

Figure 30 – Difference between ERA based on $\mathring{e}_a^{M_s}$ and ERA in 2100 by subregions



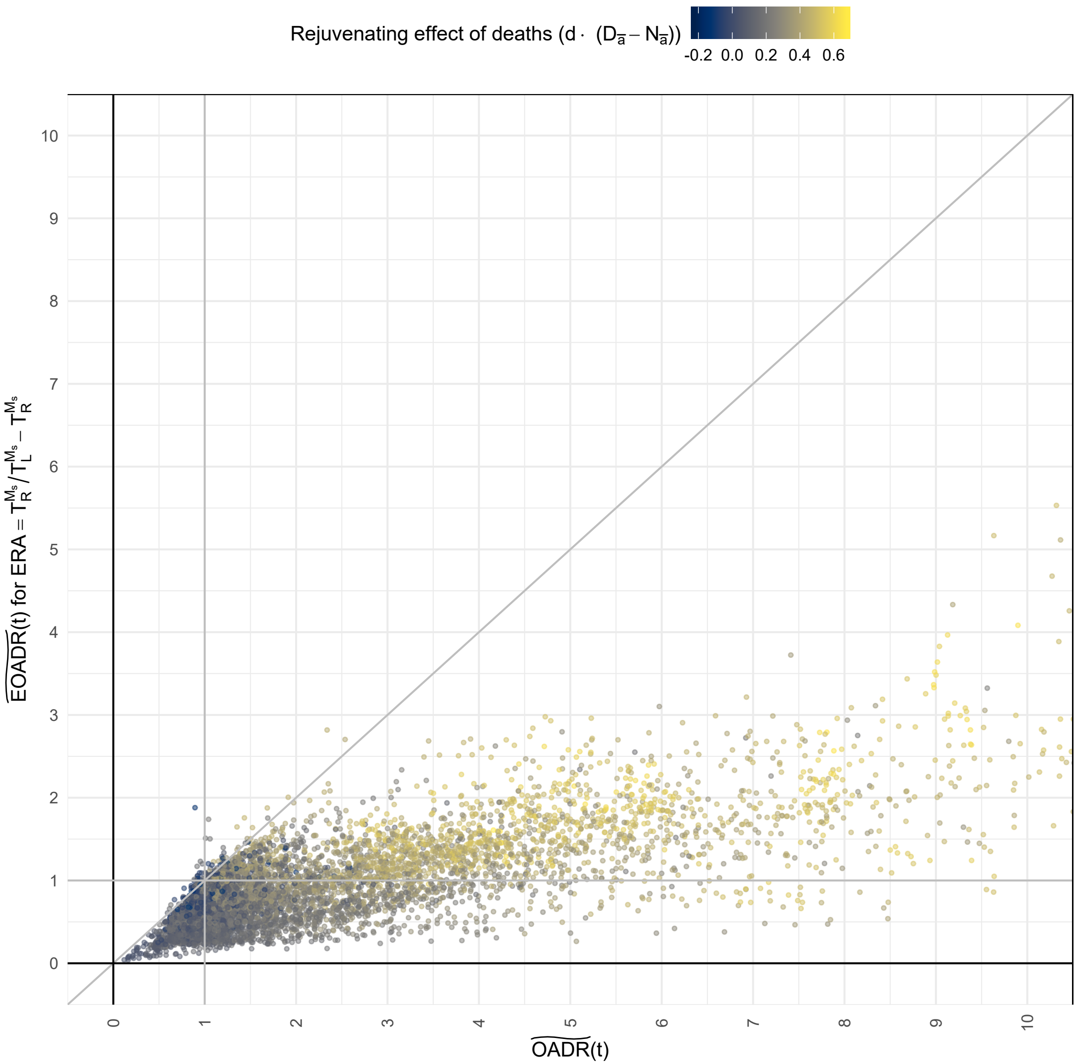
Source: Authors' calculations, based on United Nations (2017b).

Notes: Equivalent retirement age (ERA) = $(T_R)/(T_L - T_R)$. Equivalent retirement age (ERA) based on $\mathring{e}_a^{M_s} = (T_R^{M_s})/(T_L^{M_s} - T_R^{M_s})$. Square indicates the mean of the distribution.

We plot $\widetilde{OADR}(t)$ by $\widetilde{EOADR}(t)$ in Figure 31.[27] Equivalent retirement ages (ERA) based on $\mathring{e}_a^{M_s}$ are generally higher than ERA based on $\mathring{e}_a$; consequently, $\widetilde{EOADR}(t)$ for ERA based on $\mathring{e}_a^{M_s}$ are mostly lower than $\widetilde{EOADR}(t)$ for ERA based on $\mathring{e}_a$. Therefore, if the OADR is aging relatively to ERA's base year, we may expect that ERA based on $\mathring{e}_a^{M_s}$ increase the number of populations that over-compensate and decrease the number of populations that under-compensate. We group the number of countries by ERA based on $\mathring{e}_a^{M_s}$ effectiveness category, and rejuvenating effect of deaths interval and respective stage of the demographic transition in Table 8. For a total of 6,030 populations, ERA based on $\mathring{e}_a^{M_s}$ maintains over-rejuvenate at 19% (1,173 to 1,141), increases over-compensate from 30% to 36% (1,794 to 2,191), holds effectively compensate at 6% (353 to 380), and decreases under-compensate from 44% to 37% (2,634 to 2,205), with from 1% to 2% (76 to 113) in the remaining categories. The most relevant changes happen for populations that have rejuvenating effects of deaths between 0.2 and 0.6 (stage 4 of the demographic transition). For 1,925 populations that have rejuvenating effects of deaths between 0.2 and 0.4 (i.e., first half of stage 4), over-compensate increases from 32% to 45% (625 to 871), effectively compensate virtually remains the same from 10% to 11% (194 to 217), and under-compensate decreases from 52% to 38% (999 to 733). For 1,383 populations that have rejuvenating effects of deaths between 0.4 and 0.6 (i.e., second half of stage 4), under-compensate decreases 96% to 87% (1,326 to 1,209). Therefore, although ERA based on $\mathring{e}_a^{M_s}$ increase the number and level of over-compensate, and decrease the number and level of under-compensate (see Figures 22 and 31), for rejuvenating effects of deaths above 0.4, they improve the likelihood to effectively compensate.

[27] Figure 34 in Appendix — Equivalent retirement ages based on the modal age at death details Figure 31 by subregions.

Figure 31 – $\widetilde{OADR}(t)$ by $\widetilde{EOADR}(t)$ for ERA based on $\mathring{e}_a^{M_s}$



Source: Authors' calculations, based on United Nations (2017b).
Note: Equivalent retirement age (ERA) = $(T_R^{M_s})/(T_L^{M_s} - T_R^{M_s})$.

Table 8 – Number of countries by equivalent retirement age (ERA) based on $\mathring{e}_a^{M_s}$ effectiveness category, and rejuvenating effect of deaths interval and respective stage of the demographic transition

Equivalent retirement age (ERA) effectiveness category	Rejuvenating effect of deaths								Total
	−0.2, 0.0	0.0	0.0, 0.2	0.2	0.2, 0.4	0.4, 0.6	0.6	> 0.6	
	Stage of the demographic transition								
	1	1A	2	3	4	4	4A	5	
Neutral-aging	5	1	9	3	12	1	0	0	31
Over-neutralize	3	2	3	0	0	0	0	0	8
Effectively neutralize	4	0	4	1	0	0	0	0	9
Under-neutralize	10	2	6	4	0	0	0	0	22
Over-rejuvenate	122	80	778	78	83	0	0	0	1,141
Over-compensate	52	59	791	320	871	98	0	0	2,191
Effectively compensate	7	10	64	16	217	66	0	0	380
Under-compensate	18	7	107	51	733	1,209	62	18	2,205
Overload aging	8	1	11	5	9	9	0	0	43
Total	229	162	1,773	478	1,925	1,383	62	18	6,030

Source: Authors' creation and calculations, based on United Nations (2017b).

Notes: Equivalent retirement age (ERA) = $(T_R^{M_s})/(T_L^{M_s} - T_R^{M_s})$. Neutral-aging = 0.95 =< $\widetilde{OADR}(t)$ <= 1.05 and 0.95 =< $\widetilde{EOADR}(t)$ <= 1.05; Effectively compensate = $\widetilde{OADR}(t)$ > 1.05 and 0.95 =< $\widetilde{EOADR}(t)$ <= 1.05; Effectively neutralize = $\widetilde{OADR}(t)$ < 0.95 and 0.95 =< $\widetilde{EOADR}(t)$ <= 1.05. The other ERA effectiveness categories are adjusted accordingly. Stage of the demographic transition determined exclusively by rejuvenating effect of deaths $(d(t) \cdot [D_{\overline{a}}(t) - N_{\overline{a}}(t)])$.

CONCLUSION

Population aging is unavoidable, pervasive, and an uninsurable risk to any pay-as-you-go (PAYG) retirement system because it changes the relation between beneficiaries and contributors. An alternative policy that may buffer the burden of population aging in PAYG systems is to adjust the age of entry into retirement (R) based on gains in life expectancy, also known as equivalent retirement age (ERA). This policy has been implemented by several countries across the globe. Nevertheless, we argue that equivalent retirement ages (ERA) are intrinsically ineffective because PAYG systems are structured on population characteristics, while equivalent retirement ages (ERA) are based on life cycle characteristics. We propose effectiveness categories for equivalent retirement ages (ERA) that are based on the change of both the old age dependency ratio (OADR) and the equivalent old age dependency ratio (EOADR) relatively to ERA's base year. We demonstrate that if the old age dependency ratio (OADR) is aging relatively to the equivalent retirement age (ERA)'s base year, on the one hand, the lower the rejuvenating effect of deaths, the higher the probability that the equivalent retirement age (ERA) increases the age of entry into retirement (R) more than necessary; on the other hand, the higher the rejuvenating effect of deaths, the higher the likelihood that the equivalent retirement age (ERA) increases the age of entry into retirement (R) less than necessary. Also, we argue that when old-age mortality declines, equivalent retirement ages (ERA) based on gains in the modal life expectancy at age a ($\mathring{e}_a^{M_s}$) may be less ineffective than equivalent retirement ages (ERA) based on gains in the life expectancy at age a ($\mathring{e}_a$).

From a policy guidance standpoint for PAYG systems that adopt equivalent retirement ages (ERA), if the old age dependency ratio (OADR) is not aging relatively to the equivalent retirement age (ERA)'s base year, policymakers should not increase the age of entry into retirement (R). If the old age dependency ratio (OADR) is aging and the equivalent old age dependency ratio (EOADR) is not aging both relatively to the equivalent retirement age (ERA)'s base year, policymakers should either increase the age of entry into retirement (R) to less than determined by the equivalent retirement age (ERA), or increase the age of entry into retirement (R) to the age bounded by the equivalent retirement age (ERA) and also decrease the contribution rate (*con*), increase the benefit rate (*ben*), or both. If the old age dependency ratio (OADR) and the equivalent old age dependency ratio (EOADR) are both aging relatively to the equivalent retirement age (ERA)'s base year, policymakers should increase the age of entry into retirement (R) to more than delimited by the equivalent retirement age (ERA), or increase the age of entry into retirement (R) to the age regulated by the equivalent retirement age (ERA) and increase the contribution rate (*con*), decrease the benefit rate (*ben*), or both. Fixed relative position (FRP) PAYG systems have the best policy design to face any of these scenarios, not only because they offer greater flexibility for *ad hoc* policy changes to contribution rates (*con*) or benefit rates (*ben*), but because they dilute any risk of equivalent retirement age (ERA) ineffectiveness between contribution rates (*con*) and benefit rates (*ben*) as well.

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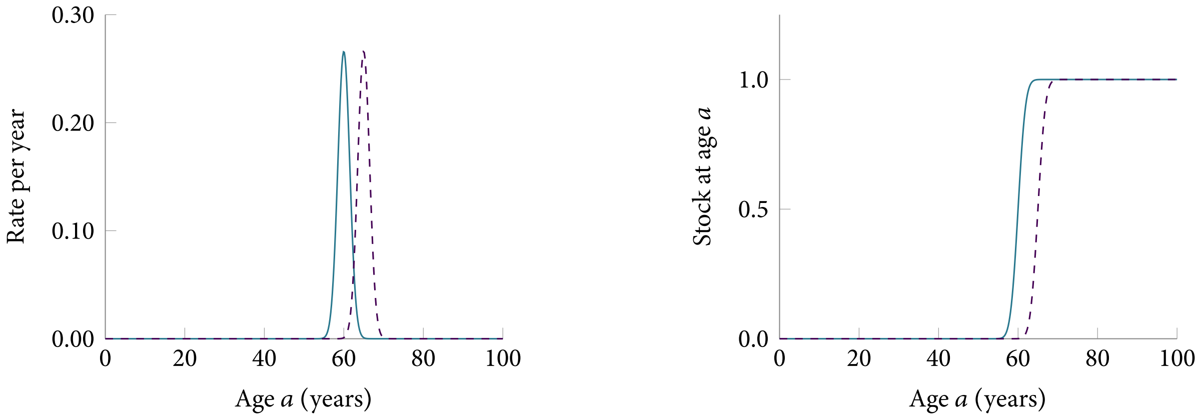
APPENDIX

APPENDIX — AGE REBALANCES: EQUIVALENT RETIREMENT AGES

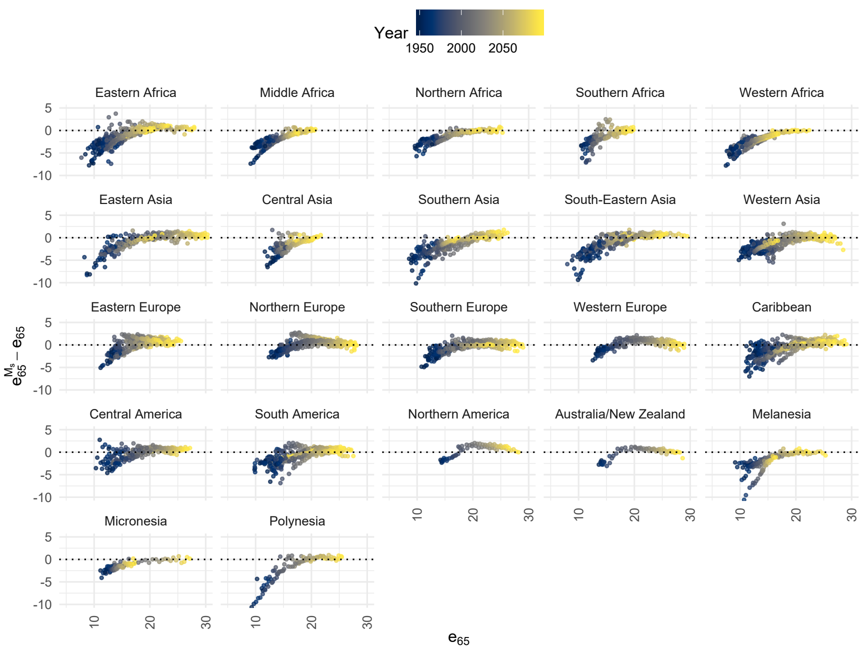
Figure 32 – Proportional rescaling of the age of entry into retirement from 60 years to 65 years under weak proportionality



Source: Authors' creation, based on Figure 1 in Lee and Goldstein (2003, Supplement, p. 186).
Note: Curves are for variance (σ^2) equal to 1.5 years.

APPENDIX — EQUIVALENT RETIREMENT AGES BASED ON THE MODAL AGE AT DEATH

Figure 33 – Life expectancy at age a ($\mathring{e}_a$) by modal life expectancy at age a minus life expectancy at age a ($\mathring{e}_a^{M_s} - \mathring{e}_a$) and subregions for age 65



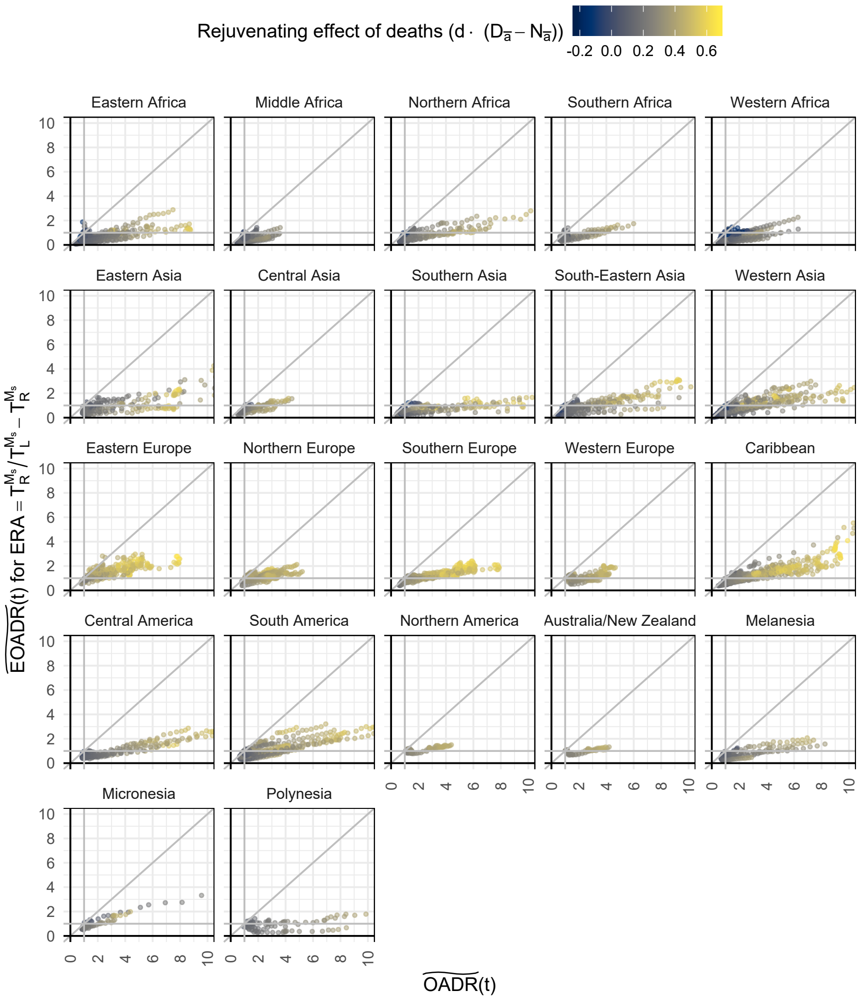
Source: Authors' calculations, based on United Nations (2017b).

Table 9 – Equivalent retirement age (ERA) by point of measurement and characteristic of measurement based on $\mathring{e}_a^{M_s}$

Point of measurement	Characteristic of measurement			
	Modal expected years in retirement		Ratio of modal expected years in retirement to modal expected years in work	
Entry into retirement (R)	$\mathring{e}_R^{M_s}$	(23)	$\dfrac{\mathring{e}_R^{M_s}}{R-L}$	(24)
Entry into labor force (L)	$\dfrac{l_R}{l_L} \cdot \mathring{e}_R^{M_s}$	(25)	$\dfrac{l_R / l_L \cdot \mathring{e}_R^{M_s}}{\mathring{e}_L^{M_s} - l_R / l_L \cdot \mathring{e}_R^{M_s}}$	(26)

Source: Authors' creation, based on table in Bayo and Faber (1981, p. 4).

Figure 34 – $\widehat{OADR}(t)$ by $\widehat{EOADR}(t)$ for $(T_R^{M_s})/(T_L^{M_s} - T_R^{M_s})$ and subregions



Source: Authors' calculations, based on United Nations (2017b).