Medical progress as a driver of (unequal) life cycle outcomes*

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Abstract

Motivated by the fact that within-cohort inequality in wealth and in life expectancy increase over the life cycle, we propose a normative framework for studying how heterogeneous individuals, who differ by ability and initial health conditions, accumulate human capital, assets, social security wealth, and health deficits over the life cycle. To do so, we implement a life cycle model in which individuals face mortality risk and optimally decide about their education, consumption, their labor supply (intensive and extensive margins), and on their health care expenditure, which is used to reduce the speed of accumulation of health deficits and hence their risk of dying. Based on a calibration for the US, we study how productivity growth and medical progress bear on the life-cycle behaviors and outcomes of set of birth cohorts ranging between 1910 and 1970. We identify a key role for medical progress in driving increases in health care spending and life expectancy as well as for the expansion of education and the reversal from an initial tendency towards earlier retirement to a postponement. We also find that both productivity growth and medical progress contribute considerably to growing inequality.

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1 Introduction

Following years of seemingly unabated and universal growth in economic prosperity as well as in life expectancy in developed economies, two challenges emerged over the past decade following the aftermath of the 2008 recession. First, many economies are struggling to sustain their welfare states which have now come under the twin pressure of population aging and austerity policies. Second, in many countries inequality is reemerging as a policy issue, with inequality relating to income and wealth as well as to health and longevity (Autor et al., 2008; Atkinson et al., 2011; Chetty, 2016; Saez and Zucman, 2016; Case and Deaton, 2017; OECD, 2017).

Here, the recognition that people are typically most vulnerable during their childhood and during old age - and in particular so in societies in which family ties are weakening - has triggered a specific interest in the evolution of inequality over the life course (e.g. National Academies of Sciences, Engineering, and Medicine (2015) and OECD (2017)). A thorough understanding of the life-cycle dynamics of inequality is important for a number of reasons: First, it is important to recognize the multi-dimensionality of inequality (education, wealth, health, living conditions, etc.) and the fact that from a life-course perspective many of these "factors" can be seen both as drivers and as outcomes, depending on the particular stage of the life-cycle. Early-life health, for instance, determines educational outcomes (e.g. Bleakley, 2007, 2010; Field et al., 2009), which jointly determine mid-life health and income (e.g. Smith, 2007; van Kippersluis et al., 2010). By contrast, education and mid-life income determine later life health. Second, the progression of (inequality in) life-cycle outcomes follows from the interaction of two processes: (i) the sequential transmission of initial early-life inequality into later-life cycle behaviors and ultimately into later-life outcomes, and (ii) the accumulating impact(s) of a process of (correlated) shocks to earnings, employment and health over the life-cycle. While the former could be understood as a form of fundamental inequality that should be addressed by redistributive policies, the latter can be understood to be a matter of insurance rather than (re-)distribution. Third, moving from an individual (or cohort) life-cycle perspective to a population (or cross-cohort) perspective, the emergence of life-cycle patterns of inequality over time is prone to be subject to two forces: changes in socio-economic conditions due to e.g. productivity growth or medical progress, which are prone to translate into (i) different starting conditions and life-cycle dynamics for successive cohorts and (ii) into different (early-life) incentives for members of successive cohorts to select into certain socio-economic sub-groups, e.g. by choice of an educational pathway, which in and of themselves induce changing life-cycle patterns and outcomes. A thorough understanding of these processes and the resulting life-cycle patterns and outcomes is a crucial prerequisite for policy-making, not the least as policy interventions themselves will shape the emergence of life-cycle outcomes depending on which stage of the life-course they are targeted at.

In this paper we seek to gain a deeper understanding of the behavioral incentives and resulting life-cycle dynamics within the nexus of health, education, labour supply and wealth, how they vary with an individual's endowment in terms of health and ability, and how the resulting patterns of increasing inequality emerge over time depending on (i) productivity growth and (ii) medical progress.

For reasons of tractability and in order to focus on the role of "fundamental" heterogeneity rather than life-cycle risks, we abstract in this study from random shocks over the life-cycle, leaving only survival to follow a random process. We motivate this by pointing to the seminal study by Huggett et al. (2011) who show that initial heterogeneity (ability to learn, human capital, wealth at age 23) rather than random shocks explain about 61.2, 62.4 and 66.0 percent of the variation in US lifetime earnings, lifetime wealth and lifetime utility, respectively. This is consistent with earlier findings by Cunha et al. (2016) that about 60 percent of earnings variability is foreseen by agents and cannot therefore be attributed to uncertainty.

Specifically, we model the life-cycle for a succession of birth cohorts in an economy with ongoing productivity growth and medical progress, the latter rendering health care more effective. Members of each cohort are drawn from a time-constant distribution in regard to the initial endowment, consisting of the initial number of health deficits, an (effort) cost of schooling, and ability to convert education into human capital. Individuals face three stages of their life-course —education, working life, and retirement— the length of these stages being determined endogenously through the initial selection into an education pathway which determines the duration of schooling and through the choice of a retirement age and (implicitly) a maximum duration of life. In addition, individuals decide on a consumption path; a path of health investments, determining the accumulation of health deficits; and, during their working age, a path of labour supply. In so doing, they maximize their expected life-cycle utility, the mortality risk being determined by the number of health deficits. By considering a large number of random draws from the initial distribution for each cohort (taken at 10 year intervals), we obtain distributions of life-cycle outcomes for each of three educational categories (primary, secondary, post-secondary). Then allowing self-selection into education of individuals according to their initial endowment, we can model life-cycle outcomes by educational category. We calibrate the model to US data trying to replicate developments of key life-cycle indicators, such as the emergence of longevity, the spending share on health care, retirement age, life-cycle income, and the distribution of education over time. Our model then allows a more detailed analysis of the various pathways by which the emergence of the various indicators within and across education categories trace to the individual's endowment as well as to the socio-economic environment in terms of productivity and state of medicine. We round-off our analysis by studying two counterfactual scenarios, in which we alternately shut down productivity growth and medical progress. This allows us to identify the role of these trends in shaping the emergence of the distribution of life-cycle outcomes over time.

Our benchmark analysis traces very well the data on the evolution of educational structure, retirement, life-expectancy, the health care spending share and the value of life for cohorts born between 1910 and 1970 into the US economy and yields the following insights. The increase in life expectancy, afforded by ongoing medical progress, together with productivity growth translates into an increasing incentive for individuals with a given endowment to select successively into higher modes of education. This provides a further boost to income, which in turn raises the value of life and, thus, the incentive to spend on health care. Indeed, the health care spending share is growing very much in line with the literature. There is marked heterogeneity in health care spending across education groups, with the spending share of high-educated individuals with high earnings increasing much more strongly. Together with medical progress this translates into a widening gap in life-expectancy.

Both the increase in life-expectancy and productivity growth translate into an incentive to postpone retirement. This is offset, however, by an income effect calling for an earlier retirement in order to enjoy more leisure. This effect dominates for early-born cohorts who do not benefit yet too much from an expansion of longevity. For these cohorts the only way to expand their retirement stage is to retire earlier. A reversal occurs for later-born cohorts who experience a strong expansion of their life-time and who can therefore afford to postpone retirement in order to allow for greater consumption during old age. Again, there is a considerable divergence in retirement age across education groups, with the trend towards a postponement of retirement setting in much later for the lesser-educated. This leads to a widening in the retirement gap. Strikingly, these divergent life-cycle trends do not result in an increase in the inequality with respect to the value of life at age 14 as a measure of life-cycle utility.

Our counterfactuals highlight both productivity growth and medical progress as important drivers of the evolution and heterogeneity of life-cycle outcomes.

While the transition into higher education, the trend towards a postponement of retirement, health care spending growth and the increase in life-expectancy tend to be weakened in the absence of productivity growth, the ongoing presence of medical progress continues to drive these life-cycle developments. By contrast, a shutting down of medical progress leads to rather different outcomes. Stagnating medical effectiveness translates into both a stagnation of the health care spending share and a stagnation in life-expectancy. Strikingly, there is now an "adverse" selection into lower education groups, which is partly explained by the fact that the benefits from productivity growth indiscriminately accrue to all education groups. Given that health care spending does not become more effective, income gains are transformed predominantly into consumption and leisure. This implies an unbroken trend towards earlier retirement. Altogether, this suggests a pivotal role of medical progress for explaining observed life-cycle outcomes, as has not been identified before.

Our modeling combines various strands of the literature. First, it relates recently developed life-cycle models of health and health behavior which are based on a biologically plausible mechanism of deficit accumulation (e.g. Mitnitski et al., 2002a,b; Dalgaard and Strulik, 2014, 2017; Schünemann et al., 2017; Dragone and Strulik, 2018; Strulik, 2018) to a literature which uses the statistical value of life as a measure that governs individual incentives to invest in their health (e.g. Murphy and Topel, 2006; Hall and Jones, 2007; Kuhn et al., 2015). In this contribution we derive in a rigorous way the value of life within a life-cycle model of deficit accumulation, show how they relate to each other, and study how they govern incentives to invest in health and longevity. In addition, we broaden the scope to include retirement and education choices both at the individual level, as studied e.g. by Cervellati and Sunde (2013) and Sánchez-Romero et al. (2016) but without the health dimension.[1] In contrast to all of the named models, we apply our model to examine systematically how the resulting life-cycles vary with the individuals' endowments in terms of health deficits, ability and disutility of schooling effort.[2]

Second, in this last aspect our paper relates to the literature that explores the heterogeneity of life-cycle behaviors and outcomes.[3] While structural mod-

[1] d'Albis et al. (2012) is a prequel in the sense of examining the impact of exogenous changes in life-expectancy on retirement decisions alone. Kuhn et al. (2015) and Dalgaard and Strulik (2017) examine the interaction of the demand for health care and retirement choices but both papers disregard education.

[2] Strulik (2018) explores the education gradient in health and health behavior but does not develop the full nexus of inequality.

[3] By additionally, including occupational choices and health-related life-styles, Galama and van Kippersluis (2018) formulate an even richer theory of a socio-economic gradient

els typically focus on the impact of health and earnings shocks on the accumulation of wealth over the life-cycle (e.g. de Nardi et al., 2010; Kopecky and Koreshkova, 2014; Capatina, 2015; Laitner et al., 2018), we focus on the way in which initial heterogeneity translates into later-life inequality both through the (early) selection into educational groups (and by implication into different career paths) and through the adoption of divergent life-cycle behaviors in respect to savings, health investments, labour supply, and retirement. As such, our work is more akin to Sánchez-Romero and Prskawetz (2017), studying how initial differences in ability and frailty translate into differing accumulation of human capital and financial wealth depending on the pension scheme; and Frankovic and Kuhn (2019), studying how differences in initial skills/education translate into differences in the utilization of innovative health care and resulting longevity.

Finally, in studying the long-run implications for life-cycle behaviors and outcomes, especially in relation to health care spending and longevity, of productivity growth as opposed to medical progress our work contributes to a literature seeking to identify the contributions of these factors to the simultaneous boost in health care spending and longevity over the past decades (e.g. Hall and Jones, 2007; Fonseca et al., 2013; Jones, 2016b; Böhm et al., 2018; Frankovic and Kuhn, 2018). In contrast to these models, we do not only depict the impact of these changes not only on the mean but also on the distribution both across and within educational sub-groups.

The remainder of the paper is organized as follows. Section 2 introduces the model, derives the optimal allocation and discusses some of the salient pathways; Section 3 deals with the calibration of the model; Section 4 presents our findings; and Section 5 concludes.

2 The model

2.1 Individual behavior

We build our model by connecting the literature on the impact of longevity improvements on schooling and retirement by Sánchez-Romero et al. (2016) and the willingness to pay for health and its impact on retirement by Kuhn et al. (2015) with the health deficit model developed by Dalgaard and Strulik

in health based on the Grossman (1972) style model of a health stock. While their work identifies many similar incentives analytically, they abstain from a full-blown calibration and numerical analysis.

(2014). Moreover, we assume heterogeneous individuals that differ with respect to their learning ability level, initial health deficits, and schooling effort.

Budget constraint. Consider an individual who faces a mortality risk as in Yaari (1965). Given a set of a priori endowments on learning ability, θ, initial health deficits, $D(0)$, and schooling effort, ϕ, the individual chooses the consumption path, $c(t)$, health care, $h(t)$, the fraction of time devoted to work, $\ell(t)$, the length of schooling, E, and the retirement age, R, that maximize a lifetime utility V. As we are considering a partial equilibrium setting and do not aggregate across cohorts, we can interpret the variable t as both age and time. The individual accumulates health deficits $D(t)$ which tend to increase the mortality risk (Mitnitski et al., 2002a,b). We assume that a number of deficits $D(t) \geq \overline{D}$ is entirely incompatible with survival and we denote by T the age at which $D(t) = \overline{D}$. Health care investments slow down the accumulation of health deficits and hence reduce mortality and postpone age T.

The life span of the individual is divided into three stages: schooling, working life, and retirement. In the first stage (= schooling) the individual chooses consumption, health care, and the number of years of schooling, E. Additional schooling increases the future wage rate per hour worked according to a standard Ben-Porath (1967) mechanism

$$H(E|\theta) = e^{-\delta E}\left(H_0 + (1-\gamma)\theta\int_0^E e^{\delta(1-\gamma)t}dt\right)^{\frac{1}{1-\gamma}}, \tag{1}$$

where H_0 is the initial stock of human capital, which is normalized to one, $\delta \geq 0$ is the human capital depreciation rate, $0 < \gamma < 1$ is the returns to education, and $\theta > 0$ is the innate ability of the individual. However, attaining a higher educational level comes at the expense of two costs: First, a disutility from the effort of attending school, where the effort accounts for the socioeconomic environment and other non-monetary factors that tend to deter the individual from attaining education as well as for a possible myopic lack of foreseeing the returns of higher education (Oreopoulos and Savanes, 2011). Second, the present value of the income stream that is foregone during the time spent on education.

In the second stage (= working life), the individual chooses consumption, health care, the number of hours worked in exchange of a wage rate, and the amount of savings necessary to finance the expenditures during retirement. In the third stage (= retirement), the individual consumes the stock of capital on consumption goods and health care and enjoys leisure.

To rule out adverse selection of health investment on retirement and mortality, as in Kuhn et al. (2015), and in the spirit of much of the life-cycle literature, we assume individuals do not purchase annuities. Moreover, we assume the individual starts and dies without wealth, i.e. $k(0) = k(T) = 0$. Thus, the lifetime budget constraint is

$$\int_0^T e^{-rt} \left(c(t) + p_h(t)h(t) + p_m(t)\mu(t)\right) dt = \int_E^R e^{-rt} w(t, E|\theta)\ell(t)dt, \qquad (2)$$

where r is the market interest rate, $p_h(t)$ is the price of health care goods at time t, and $p_m(t)\mu(t)$ is the expenditure on emergency care required at time/age t. We assume emergency care to be proportional to the mortality hazard, $\mu(t)$, at age t, the price $p_m(t)$ then reflecting the appropriate conversion rate into money. All consumption goods and health care costs are financed by the labor income earned during the working period between ages E and R. The labor income earned at age t is given by the labor supply, $\ell(t)$, at this age and the wage rate $w(t, E|\theta)$.

Preferences. The expected lifetime utility at time 0, conditional on the years of schooling, E, longevity, T, retirement age, R, consumption path, c, employment path, ℓ, and health care path, h, is given by

$$V(E, T, R, c, \ell, h) = \int_0^T e^{-\rho t} S(t) u(c(t), \ell(t)) dt - \phi \int_0^E e^{-\rho t} S(t) dt + \int_R^T e^{-\rho t} S(t)\varphi(t) dt. \qquad (3)$$

The first term on the right-hand side accounts for the utility of consumption and the disutility of work (with $u(c, \ell) > 0$, $u'_c(c, \ell) > 0$, $u'_\ell(c, \ell) < 0$, $u''_{cc}(c, \ell) < 0$, $u''_{\ell\ell}(c, \ell) < 0$, and $u''_{c\ell}(c, \ell) \leq 0$). The second term reflects the effort of attending school (Sánchez-Romero et al., 2016; Restuccia and Vandenbroucke, 2013; Oreopoulos, 2007), which is an increasing function of the length of schooling, E. The last term captures the utility of leisure during retirement, where $\varphi(t) > 0$ is the marginal utility of leisure during retirement (Sánchez-Romero et al., 2019). We assume $\varphi'(t) > 0$; i.e., the marginal utility of leisure in retirement increases with age as the remaining life-time in retirement is squeezed.

Survival and health deficit accumulation. To connect the health deficits model of Dalgaard and Strulik (2014) to the conventional value of life approach

(Murphy and Topel, 2006; Hall and Jones, 2007; Kuhn et al., 2011, 2015), we follow Schünemann et al. (2017) and Dragone and Strulik (2018) and introduce survival, $S(t)$, as a distinct third state variable, which is subject to the dynamics

$$\dot{S}(t) = -\mu(D(t))S(t). \tag{4}$$

$\mu(D(t))$ is the instantaneous mortality (hazard) rate at age t, which is a function of the state variable: stock of health deficits at time t, $D(t)$. According to Mitnitski et al. (2002a,b), the stock of health deficits, $D(t)$, measures the proportion of possible deficits an individual has at age t. Therefore, by construction $D(t)$ takes values between zero and one. Moreover, Mitnitski et al. (2002a) show that the instantaneous mortality rate can be expressed as a function of health deficits according to

$$\mu(D(t)) = \gamma_\mu + \alpha_\mu \left(\frac{D(t) - \gamma_d}{\alpha_d}\right)^{\frac{\beta_\mu}{\beta_d}} \text{ with } D(0) \geq \gamma_d + \alpha_d, \tag{5}$$

where $\{\alpha_\mu, \beta_\mu, \gamma_\mu\}$ and $\{\alpha_d, \beta_d, \gamma_d\}$ are positive parameters associated with a Gompertz–Makeham law on mortality and on health deficits, respectively.[4] Eq. (5) shows how an increase in health deficits raises the mortality hazard rate. Following Dalgaard and Strulik (2014) we assume health deficits to accumulate with age according to a natural rate of aging, β_d, which can be slowed down by investing in health, $h(t)$. Thus, the stock of health deficits evolves at age t according to

$$\dot{D}(t) = \beta_d \left(D(t) - A(t)h(t)^\eta - \gamma_d\right) \text{ with } D(T) = \overline{D}, \tag{6}$$

where $\overline{D}$ is the maximum stock of health deficits compatible with being alive, η measures the returns to health investments, which is assumed to range between zero and one, and $A(t)$ represents the state of the health technology at time t. For simplicity, we assume the state of health technology to increase exponentially at a constant rate g_h; i.e. $A(t) = A(0)e^{g_h t}$. From (5) and (6) we can rewrite the mortality hazard rate in terms of the Makeham–Gompertz law on mortality and of health investments as

$$\mu(t) \equiv \gamma_\mu + \alpha_\mu \exp\left\{\beta_\mu t + \frac{\beta_\mu}{\beta_d}\log(1 - \mathrm{Re}(t))\right\}, \tag{7}$$

where

$$\mathrm{Re}(t) = \frac{\beta_d}{\alpha_d}\int_0^t A(0)h(s)^\eta e^{-(\beta_d - g_h)s} ds, \tag{8}$$

[4] Mitnitski et al. (2002a) obtain that more than 99% of the total variance in mortality rates is explained by combining these two Gompertz-Makeham laws.

with $0 \leq \text{Re}(t) < 1$, represents the rate of "rejuvenation" at age t, amounting to the compounded health-care-driven deficit reduction up to this age. Taken together, eqs. (7) and (8) imply that as the individual accumulates health care investments, the speed of aging, or senescence, is slowed down. By differentiating (7) with respect to $\text{Re}(t)$, we obtain

$$\frac{\partial \mu(t)}{\partial \text{Re}(t)} = -\frac{\beta_\mu(\mu(t) - \gamma_\mu)}{\beta_d(1 - \text{Re}(t))} < 0. \tag{9}$$

Thus, the impact of the rejuvenation rate on the reduction of mortality is greater the higher is the mortality (hazard) rate and the greater is the level of the rejuvenation. However, since $\lim_{t\uparrow\infty} \text{Re}'(t) = (\beta_d/\alpha_d)A(0)h(t)^\eta e^{-(\beta_d - g_h)t} = 0$, for $\beta_d > g_h$, the rejuvenation rate will reach a plateau at old age and there will not be further reductions in mortality.

An important feature of eqs. (7) and (8) is that they allow us to derive the overall impact on mortality of an increase in the state of the medical technology. Differentiating (7) with respect to the initial state of medical technology, $A(0)$, we obtain

$$\frac{\partial \mu(t)}{\partial A(0)} = \frac{\partial \mu(t)}{\partial \text{Re}(t)} \frac{\text{Re}(t)}{A(0)} \left(1 + \eta \varepsilon_{A(0),h}(t)\right). \tag{10}$$

where $\varepsilon_{A(0),h}(t)$ is the average elasticity of health care investments in response to medical technology at time t. [5] Thus, a higher state of medicine reduces the mortality rate, i.e. $\frac{\partial \mu(t)}{\partial A(0)} < 0$, if health care investments increase with medical technology on average or at least do not decline by too much, such that $\varepsilon_{A(0),h}(t) > -1$. In this case, $\text{Re}'(t) > 0$ and $\frac{\partial}{\partial t}(\frac{\partial \mu(t)}{\partial \text{Re}(t)}) < 0$ imply that better medical technology will reduce mortality disproportionately at higher ages.

2.2 Optimal life-cycle allocation

The problem of the individual is to maximize (3) subject to (1)–(2) and the boundary conditions on health deficits $D(0) = D_0$, $D(T) = \overline{D}$ and on capital $k(0) = k(T) = 0$. To solve the individual problem numerically we assume an additively separable instantaneous utility function in consumption and labor

$$u(c, \ell) = \left(c^{1-\sigma_c^{-1}}\right) / \left(1 - \sigma_c^{-1}\right) - \alpha_l \left(\ell^{1+\sigma_l^{-1}}\right) / \left(1 + \sigma_l^{-1}\right) + \tilde{u}, \tag{11}$$

[5] The elasticity is defined as $\varepsilon_{A(0),h}(t) := \int_0^t \left(\frac{A(0)}{h(s)} \frac{\partial h(s)}{\partial A(0)}\right) \frac{A(s)h(s)^\eta e^{-\beta_d s}}{\int_0^t A(\tau)h(\tau)^\eta e^{-\beta_d \tau} d\tau} ds$.

where $0 < \sigma_c$, $\sigma_l < 1$ are the intertemporal elasticities of substitution (IES) on consumption and labor, respectively; where $\alpha_l \geq 0$ is the weight of the disutility of labor; and where $\tilde{u} > 0$ is a baseline benefit from being alive. Its value is assumed to be large enough to guarantee that $u(c, \ell) > 0$ holds for all ages, which guarantees a willingness to survive throughout.[6]

We derive the optimal choices of the individual in four steps. First, conditional on a particular length of schooling, E; retirement age, R; and maximum age, T, we obtain the optimal consumption path, health care investments, and hours worked. Second, based on the conditional choices of the optimal controls, the length of schooling, and the maximum age, we obtain the optimal retirement age. Third, we derive the optimal maximum age conditional on the optimal controls and the optimal retirement age for any given length of schooling. Finally, we calculate the length of schooling that maximizes the lifetime utility. A full derivation of the individual problem can be seen in Appendix A.

For simplicity' sake, we define $\mathbf{u}(\cdot)$ such that

$$\mathbf{u}(t) \equiv \begin{cases} u(c(t), 0) - \phi & \text{for } 0 < t \leq E, \\ u(c(t), \ell(t)) & \text{for } E < t \leq R, \\ u(c(t), 0) + \varphi(t) & \text{for } R < t \leq T, \end{cases} \tag{12}$$

which is always a positive function. Using the current-value Hamiltonian

$$\mathcal{H} = S\mathbf{u} + \lambda_k(rk + w\ell\mathbf{1} - c - p_h h - p_m\mu(D)) + \lambda_D\beta_d(D - Ah^\eta - \gamma_d) - \lambda_S\mu(D)S, \tag{13}$$

where $\mathbf{1}$ is an indicator function that takes the value of one for $E < t \leq R$ and zero otherwise. The necessary conditions for an optimum are:

$$S(t)c(t)^{-\sigma_c^{-1}} = \lambda_k(t), \tag{14}$$

$$S(t)\alpha_l\ell(t)^{\sigma_l^{-1}} = \lambda_k(t)w(t, E|\theta) \qquad \text{for } E < t \leq R, \tag{15}$$

$$-\lambda_D(t)\beta_d A(t)\eta h(t)^{\eta-1} = \lambda_k(t)p_h(t), \tag{16}$$

where $\lambda_k(t) > 0$ is the shadow price of capital and $\lambda_D(t) < 0$ is the shadow price of health deficits. Note that the individual problem has three state variables: capital, health deficits, and survival. Therefore, there exists a $\lambda_S(t) > 0$ that is the shadow price of the probability of surviving to time t. Condition (14) shows that consumption at time t increases the higher is the probability of surviving to time t, while consumption at time t decreases the higher is the

[6] A positive baseline benefit is important even for $\alpha_l = 0$, as we are assuming $\sigma_c < 1$. This latter assumption guarantees an increase in the health care spending share with income that is in line with the data (Hall and Jones, 2007).

monetary value of assets at time t. Substituting (14) into (15) we obtain the standard condition that the marginal rate of substitution between labor and consumption must equal the wage rate, implying that the labor supply at time t increases with the wage rate and decreases with consumption at time t. Condition (16) governs optimal health investments and is similar to that obtained by Dalgaard and Strulik (2014). When dividing through by $\lambda_k(t)$, the condition states that the marginal (monetary) value of health care at time t must be equal to its (dollar) price.

Before we proceed to analyze the dynamic characteristics of the control variables, we define the value of life and the value of health deficits. First, let us define the value of life, hereinafter VOL, at time t in the absence of an annuity market as[7]

$$\psi^S(t) := \frac{\lambda_S(t)S(t)}{\lambda_k(t)} = \int_t^T e^{-r(s-t)} \frac{\mathbf{u}(s)}{\mathbf{u}'_c(s)} ds > 0. \tag{17}$$

Eq. (17) is the monetary value at age t of the discounted stream of utility flows over the remaining life span. VOL can also be defined as the marginal rate of substitution between the probability of surviving to age t and assets. Thus, VOL is a measure of the willingness to pay for avoiding death at time t. Second, let us define the value of health deficits, from now on VOD, at time t in the absence of an annuity market as

$$\psi^D(t) := \frac{\lambda_D(t)}{\lambda_k(t)} = \psi^D(T)e^{-(r-\beta_d)(T-t)} - \\ - \int_t^T e^{-(r-\beta_d)(s-t)} \mu'(D(s))(\psi^S(s) + p_m(s))ds < 0, \tag{18}$$

VOD is the marginal rate of substitution between deficits and assets. The VOD amounts to the sum of (i) the present value of the VOD at the maximum attainable age and (ii) the present value of lost survival prospects over the whole remaining life course that arises from an additional deficit at age t. Here, the deficit induced loss in survival for each future life year is evaluated by the sum of the VOL and the price of emergency health care at that given age/time. Note that discounting takes place according to the net discount rate $r - \beta_D$, i.e. the distance between interest rate and the rate of deficit accumulation.

[7] If the individual can annuitize a fraction $\kappa \in [0, 1]$ of the assets, the value of life at time t is $\psi^S(t) = \int_t^T e^{-r(s-t)} \left(\frac{S(s)}{S(t)}\right)^\kappa \frac{\mathbf{u}(s)}{\mathbf{u}'_c(s)} ds$.

Rearranging terms in (16), we obtain a closed form solution for the optimal investment in health care:[8]

$$h(t) = \left(-\beta_d \eta \psi^D(T) \frac{A(t)}{p_h(t)}\right)^{\frac{1}{1-\eta}}. \tag{19}$$

According to (19), the demand for health care increases with the state of medical technology and with the VOD in absolute terms while it declines with the price of health care.

From the first-order conditions and the envelope conditions we obtain the laws of motion for consumption, labor, and health care:

$$\frac{\dot{c}(t)}{c(t)} = \sigma_c(r - \rho - \mu(D(t))), \tag{20}$$

$$\frac{\dot{\ell}(t)}{\ell(t)} = \sigma_l \left(\frac{\dot{w}(t, E|\theta)}{w(t, E|\theta)} + \rho - r + \mu(D(t))\right), \tag{21}$$

$$\frac{\dot{h}(t)}{h(t)} = \frac{1}{1-\eta}\left(r - \beta_d + \frac{\dot{A}(t)}{A(t)} - \frac{\dot{p}_h(t)}{p_h(t)} - \frac{\mu'(D(t))\left(\psi^S(t) + p_m(t)\right)}{-\psi^D(t)}\right). \tag{22}$$

Eq. (20) is the Ramsey-rule in the absence of annuities. For $r > \rho$, the consumption path follows an inverted U-shape, since consumption starts to decline when the mortality hazard rate becomes sufficiently high. Eq. (21) shows how hours worked evolve over the working life according to the difference between the relative change in the wage rate and the relative change in consumption. The intuition is simple. When the interest is high, the individual works harder today in order to save, whereas the individual prefers to postpone work when the discount factor is high, the mortality hazard rate is high, or when the future wage rate increases. Eq. (22) shows the evolution of the age profile of health care. Health care investments tend to increase over time (i.e. they are postponed) according to the difference between the interest rate and the rate of accumulation of health deficits, and the difference between the growth of health technology and the growth of the price of health care, but they tend to be advanced to the extent that the VOD is written off year by year. Thus, health investments tend to be realised before deficits tend to bear heavily on mortality and while that risk is still weighted with a VOL and/or causes high emergency expenditure.

[8] From (19) we obtain that the elasticity of health investments with respect to health care technology at time t as $\varepsilon_{A(t),h}(t) := \frac{A(t)}{h(t)}\frac{\partial h(t)}{\partial A(t)} = \frac{1}{1-\eta}(1 + \frac{A(t)}{\psi^D(T)}\frac{\partial(\psi^D(T))}{\partial A(t)})$. Hence, improvements in the state of medical technology tend to increase health investments at time t if the VOD increases in absolute terms or if it declines inelastically.

Optimal retirement age. Given the length of schooling, E, longevity, T, and the optimal consumption path, the optimal retirement age, R^*, satisfies the condition:

$$e^{-\rho R^*} S(R^*) \mathbf{u}'_c(0) w(R^*) = e^{-rR^*} \left((\alpha_l)^{\sigma_l}(1+\sigma_l)\varphi(R^*)\right)^{\frac{1}{1+\sigma_l}}. \tag{23}$$

The left-hand side of Eq. (23) is the marginal benefit of continued working, while the right-hand side accounts for the marginal cost of postponing retirement. d'Albis et al. (2012), Kuhn et al. (2015), and Sánchez-Romero et al. (2016) investigate how changes in the mortality rate at each age, either exogenously or through health investments, affect the optimal retirement age. They show that mortality improvements early in the working life promote early retirement, while mortality improvements after retirement lead to the postponement of retirement. To show how changes in mortality at any age x_0 affects the optimal retirement age, we totally differentiate (23) with respect to R^* and $\mu(D(x_0))$

$$\frac{dR^*}{d\mu(D(x_0))} = \frac{\frac{1}{S(R^*)}\frac{\partial S(R^*)}{\partial \mu(D(x_0))} - \frac{1}{\sigma_c}\frac{1}{c^*(R^*)}\frac{\partial c^*(R^*)}{\partial \mu(D(x_0))}}{\rho - r + \mu(D(R^*)) + \frac{1}{\sigma_c}\frac{1}{c^*(R^*)}\frac{\partial c^*(R^*)}{\partial R} + \frac{1}{1+\sigma_l}\frac{\varphi'(R^*)}{\varphi(R^*)}}. \tag{24}$$

This result is very similar to that shown in d'Albis et al. (2012) and Sánchez-Romero et al. (2016) with annuities. However, there are two important differences in our model with respect to the previous ones. First, in this model mortality is endogenously chosen. Therefore, the differential age effect of the impact of mortality on retirement will be triggered by changes in the medical progress or in the technological progress. Given that these two exogenous factors affect on the mortality rate mainly at old ages, as it was shown in Eq. (10), mortality improvements will lead to late retirement (d'Albis et al., 2012) . Second, we have assumed that individuals do not purchase annuities. This assumption implies that the lifetime human wealth effect, or the positive impact of mortality on consumption due to the raise in the likelihood of receiving a future labor income stream, is shut down. Therefore, only the years-to-consume effect, which is the reduction in consumption to finance a longer lifespan, is present in the model. The years-to-consume effect has a negative impact on leisure and hence the individual delays the retirement age.

Optimal longevity and the value of health deficits. The individual reaches its possible maximum age, T, when $D(T) = \overline{D}$. To guarantee that $T = T^*$ is optimal, T must satisfy the terminal age condition, $\mathcal{H}(T^*) = 0$.

Using the terminal age condition, we derive the final VOD:

$$
\begin{aligned}
-\psi^D(T^*) = &-\frac{c(T^*)\left(1-\frac{\mathbf{u}(T^*)}{c(T^*)\mathbf{u}_c(T^*)}\right)+p_m(T^*)\mu(\overline{D})}{\beta_d(\overline{D}-\gamma_d)}+ \\
&+(-\psi^D(T^*))^{\frac{1}{1-\eta}}\frac{(1-\beta_d\eta)}{\beta_d(\overline{D}-\gamma_d)}\left(\frac{\beta_d\eta A(T^*)^{\frac{1}{\eta}}}{p_h(T^*)}\right)^{\frac{\eta}{1-\eta}}. \quad (25)
\end{aligned}
$$

A unique $\psi^D(T^*) < 0$ exists if $\eta \in (0,1)$ and if the first term on the right-hand side of (25) is negative. We then obtain three important insights from (25). First, VOD increases with total expenditure (net of health care investments). Thus, wealthier individuals invest more money than poorer individuals on health care at time T^*.[9] Second, higher prices – w, p_h, and p_m – raise the VOD. Third, an increase in the state of medical technology, $A(T^*)$, reduces the VOD at time T^*. Thus, the net effect of a simultaneous increase in labor productivity, which raises all prices including wages, and in the state of the medical technology is a priori ambiguous.

Optimal length of schooling. We assume the length of schooling is a discrete choice. This assumption has an advantage over a continuous framework, since this setting allows the possibility that the individual becomes trapped in a specific educational group when it becomes too costly to move to the next educational level.[10] Only the individual who is indifferent between two educational levels has the option of moving between levels. Hence, rather than finding an E^* that equates the marginal returns of education to the sum of the marginal costs of schooling, the individual chooses the educational level, among a set $\mathbb{E}$ of possible alternative, that reports the highest lifetime utility:

$$E^* = \arg\max_{E\in\mathbb{E}} V(E, T^*, R^*, c^*, \ell^*, h^*). \quad (26)$$

[9] When individuals purchase annuities, the consumption profile monotonically increases with age. Hence, ceteris paribus, we should expect a wider difference in the VOD at time T^* across income groups in economies in which individuals purchase annuities than in economies in which individuals do not purchase annuities. Therefore, our model assumptions produce conservative estimates of the difference in health care investments and life expectancy across income groups.

[10] Note that if the length of schooling is a continuous variable, individuals can always react to changes in the economic circumstances by marginally changing the length of schooling.

3 Calibration

We calibrate the model for the average US male born in year 1910, whose life expectancy at age 14 is 54.3 (Bell et al., 1992), the average length of schooling is 11.8 years, and the average retirement age is 64.8 (Sánchez-Romero et al., 2016).[11]

Table 1: Model parameters

Preferences			**Prices**		
IES on consumption	σ_c	0.7500	Productivity growth	g	0.0200
IES on labor	σ_l	0.2000	Rate of growth of health prices	g_h	0.0120
Utility weight of labor	α_l	15.0000	Interest rate	r	0.0400
Discount factor	ρ	0.0000	Initial price of health services	p_h	\$674
Initial utility of retirement	φ_0	3.5000	Initial price of emergency care	p_m	\$9 103
Mortality and health deficits			**Human capital**		
Natural rate of aging	β_d	0.0430	Returns to experience	β_1	0.0904
	α_d	0.0111	Returns to experience-squared	β_2	-0.0013
	γ_d	0.0200	Depreciation of human capital	δ_h	0.0011
Senescence rate	β_m	0.0768	Returns-to-scale to education	γ_h	0.6500
Minimum mortality rate	$\log(\alpha_m)$	-8.4636			
Makeham component	γ_m	0.0000	**Health investments**		
Maximum health deficits	$\overline{D}$	0.2200	Health technology	A	0.000547
			Returns-to-scale of health	η	0.2000

Similar to Dalgaard and Strulik (2017) we specify the parameters governing the accumulation of health deficits by setting the natural rate of aging (β_d) at 0.043, γ_d at 0.02, and α_d at 0.00312, which all correspond to the parameters of the health deficit function estimated by Mitnitski et al. (2002a) for Canada. Parameter η, or the returns-to-scale of health investments, is set at 0.20. The value of η is taken from Hall and Jones (2007), which deviates from the one used by Dalgaard and Strulik (2014) by 0.01. A is set at 0.000547 so that individuals born in 1910 spend 10 percent of their lifetime income on health care, which is the share of health care expenditure observed in year 1980 in the US. We choose the year in which the individual turn 70 years old, because this age corresponds to the mean-age of health care receivers observed in the US (Lee and Sanchez-Romero, forthcoming).

For consistency with the economic model, we assume the mortality rate follows a Gompertz law; i.e., $\gamma_\mu = 0$. As a consequence, we abstract from

[11] Since we analyze the behavior of individuals over their lifecycle, we use cohort data from www.ssa.gov, rather than period data from the Human Mortality Database (HMD).

the influence of infectious diseases or accidents on mortality. The minimum mortality rate, α_μ, and the senescence rate, β_μ, are set at $\exp(-8.4636)$ and 0.0768, respectively, which correspond to the estimated values of $\{\alpha_\mu, \beta_\mu\}$ for US males born in 1910 (Bell et al., 1992).

Julia/Mortality.jpeg

Figure 1: Survival probability and accumulation of health deficits: Birth cohort 1910.

We consider the wage rate per hour worked as a function of the educational attainment, conditional on the ability level, $H(E|\theta)$, the productivity growth rate g, which is assumed to increase at an annual rate of 0.02 (Jones, 2016a), and experience. Hence, the log of the wage rate per hour worked of an individual born in year i at age t, with E years of education and ability level θ, is given by

$$\log w_i(t, E|\theta) = \log H(E|\theta) + gi + gt + \beta_0 + \beta_1(t - E) + \beta_2(t - E)^2. \quad (27)$$

We consider individuals can attain any of the following three educational groups: primary, secondary, and postsecondary. To be consistent with the ISCED classification and taking into account that our model starts at age 14, we set the length of schooling, E, at 0 for primary education, at 4 for secondary education, and at 7 for postsecondary. The returns-to-education is set at $\gamma_h = 0.65$, similar to Cervellati and Sunde (2013), and δ is set at 0.0011. We differ the explanation of the calibration for θ at the end of this section. The last three Mincerian parameters in (27) are taken from Heckman et al. (2006), Table 2, which represents well the wage rate of the US cohort born in 1910 at age 30.

Prices in the health care sector are assumed to increase annually at the same rate as wages (e.g. g=2 percent). This value coincides with the growth rate of health expenditures by age for the US, as shown by Dalgaard and Strulik (2014) using data from Keehan et al. (2004). The annual growth rate of health technology, g_h, is set at 1.2 percent in order to match the increase in life expectancy at age 14 of cohorts born between year 1910 and 1970 (see Figure 2). Since the value of g_h may not necessarily represent well the reality, in Section 4 we present counterfactual experiments in which g_h is canceled out.

The parameters governing the behavior of the individual are set to replicate specific features of the consumption path and labor supply. Specifically, we assume an IES of consumption (σ_c) of 0.75, which is within the range (0.5 and 1) of values for σ_c suggested by Chetty (2006). We assume an IES of labor supply, σ_l, equal to 0.20. The weight of the disutility of labor (α) is set at 15.0 in order to obtain that prime aged individuals work 40.0 percent of their available time, which is equivalent to 44.8 hours of work per week. We set the interest rate r and the subjective discount factor at 4 percent and 0 percent, respectively, in order to match two facts. First, the observed increase by age of the cross-sectional per capita consumption profile in the US, which is around 1 percent per year. This value is calculated using the fact that cross-sectional profiles of consumption per capita grow by age at a rate equal to $\sigma_c(r - \rho) - g$. Second, an interest rate of 4 percent also gives a decline of 1 percent in the price-adjusted quality of health care at age 40 (Cutler et al., 1998), which is close to the average age of the population. The initial utility of retirement (φ_0) is set at 3.5 for individuals secondary education in order to replicate the average retirement age of US males born in 1910.

The last set of parameters corresponds to the initial endowments of our heterogeneous individuals: innate ability, θ, initial health deficits, D_0, and the

Julia/LEe14US.jpeg

Figure 2: Male life expectancy at age 14. Source: Authors' simulations and Bell et al. (1992) (red diamonds)

schooling effort, ϕ. We consider each one of these endowments is uniformly distributed within a minimum and a maximum value, which are set so as to replicate the distribution of the educational attainment of US males born in year 1910: primary=48%, secondary=43%, and postsecondary=8%. Data on educational attainment by birth cohort is taken from the project Edu20c (Goujon et al., 2016). Thus, we obtain that the schooling effort is distributed according to $\phi \sim \mathbf{u}(0.22, 0.325)$, the innate ability to $\theta \sim \mathbf{u}(0.09, 0.16)$, and the initial health deficits to $D_0 \sim \mathbf{u}(0.0235, 0.0239)$. For expositional' sake, we classify in Table 2 each endowment by low, medium, and high.

Figure 3 show how the combination of these three endowments influence on the schooling decision of our individuals. In particular, Figure 3(a) shows that completed postsecondary education (blue dots) is attained by individuals with high innate ability and low schooling effort (i.e., individuals coming from rich families). On the contrary, individuals facing a higher schooling effort (i.e.,

Table 2: Classification of initial endowments

		Low	Medium	High
Schooling effort	ϕ	(0.2200, 0.2550)	(0.2550, 0.2900)	(0.2900, 0.3250)
Innate ability	θ	(0.0900, 0.1133)	(0.1133, 0.1367)	(0.1367, 0.1600)
Initial health deficits	D_0	(0.0235, 0.0236)	(0.0236, 0.0238)	(0.0238, 0.0239)

Julia/EduByAbility&SchoolEffortB.jpeg

(a)

Julia/EduByAbility&HealthB.jpeg

(b)

Figure 3: Impact of the initial health deficit (D_0), innate ability (θ), and schooling effort (ϕ) on educational attainment. *Notes: Each panel is based on a sample of 12 500 random draws of the set of parameters.*

individuals coming from low income families), despite having a high innate ability, do not complete postsecondary education. This result is consistent with the existing report on educational attainment by ability and family characteristics (Fox et al., 2005). Looking at the influence of health on education, Figure 3(b) shows that postsecondary education is more likely to be attained individuals with high innate ability, who face low initial health deficits.

Figure 4 shows one thousand alternative optimal trajectories over the lifecycle, randomly generated with the model, for the cohort born 1910. Individuals are heterogeneous with respect to their schooling effort, innate ability, and initial health deficits. Higher profiles of labor income, consumption, value of life, and assets holding at the end of life correspond to individuals choosing postsecondary education, whereas lower per capital lifecycle profiles depict the

Julia/Profiles.jpeg

Figure 4: Life cycle profiles for different levels of innate ability, schooling effort, and initial health deficits: Birth cohort 1910.

behavior of individuals with primary education.

4 Results

We simulate three experiments. In the first experiment (our benchmark) we consider there exist medical progress, g_h, and labor-augmenting technological progress, g. In the second experiment we shut down the effect of labor-augmenting technological progress, while we assume the existence of medical progress. This experiment is named "constant prices", since all prices $\{p_h, p_m, w\}$ are assumed to depend on g. In the third experiment we assume there is no medical progress, but prices keep rising according to g. For each experiment we simulate the life cycle profiles of 20 000 individuals, that randomly differ according to their innate ability, initial health deficits, and schooling effort.

4.1 Distribution of educational attainment

Figure 5 shows in each column bar the distribution of the educational attainment of each cohort. In the benchmark model (see Fig. 5(a)), we obtain that the proportion of individuals with primary education increases from cohort 1910 to cohort 1930 and then it progressively declines. This is because individuals born before year 1940, who are indifferent between primary and secondary education, use the additional wealth to avoid the effort of attending schooling. However, individuals born after 1930 experience, on the one side, a lower schooling effort relative to the income level and, on the other, the necessity to finance the consumption over a longer life span (see Fig. 2). These two factors explain the necessity of individuals born after 1930 to invest in higher education. Thus, those individuals who remain in primary education becomes more selected over time. Table 3 reports under the benchmark scenario the evolution across cohorts of the average endowments for each educational group. We can see in Table 3 that individuals born in year 1970 who remain in primary education have (on average) low ability, average initial health deficits, and face high schooling efforts. On the contrary, the observed swing in the educational attainment between those with secondary education does not occur for individuals with postsecondary education since the latter group face lower schooling efforts and have higher returns to education (see the bottom of Table 3 for the cohort 1910).

Fig. 5(b) shows the marginal influence of the labor-augmenting technological progress on the distribution of educational attainment by birth cohort. The first important result we observe in Fig. 5(b) is that the educational attainment distribution for the cohort 1970 is lower than that in the benchmark. This is because the lack of technological progress reduces the growth rate of wages and hence the returns of education diminishes. Second, we obtain that the average educational attainment monotonically increases across cohorts, even though wages do not increase. Therefore, Fig. 5(b) suggests that medical progress at old age is a key factor explaining the increase in education. Nevertheless, since the increase in the educational distribution is greater in fig. 5(a) than in fig. 5(b), this difference suggests that rising wages complement medical

Julia/EduDist.jpeg

(a) Benchmark

Julia/EduDist01.jpeg

(b) Constant prices

Julia/EduDist02.jpeg

(c) No medical progress

Figure 5: Educational distribution: Birth cohorts 1910–1970.
Source: Authors' simulations.

progress to explain the increase in the length of schooling.

Table 3: Average values of initial endowments by educational attainment under the Benchmark: Birth cohorts 1910–1970 (in %)

Birth cohort	1910	1920	1930	1940	1950	1960	1970
Primary							
Schooling effort, ϕ	28.2	28.3	28.1	28.2	28.2	28.4	29.1
Innate ability, θ	10.9	11.0	11.1	11.0	10.7	10.3	9.8
Initial health, D_0	2.3717	2.3717	2.3723	2.3727	2.3725	2.3738	2.3774
Secondary							
Schooling effort, ϕ	26.8	26.8	26.8	27.2	27.4	27.4	27.7
Innate ability, θ	13.9	14.0	14.0	13.8	13.3	12.1	10.5
Initial health, D_0	2.3704	2.3712	2.3709	2.3713	2.3723	2.3734	2.3763
Postsecondary							
Schooling effort, ϕ	23.5	23.3	23.7	24.2	25.2	26.3	26.8
Innate ability, θ	15.1	15.1	15.1	14.9	14.6	14.1	13.1
Initial health, D_0	2.3688	2.3680	2.3661	2.3669	2.3675	2.3682	2.3699

Note: Darker colors are associated to higher values of the specific endowment. The range of values associated to each color and endowment are detailed in Table 2.

To complete the analysis with respect to education, Figure 5(c) shows how the medical progress impacts on the optimal choice of the length of schooling. By shutting down medical progress and allowing all prices to rise with the productivity growth rate, we observe in Fig. 5(c) an overall decline in educational attainment. On the one side, individuals use the rise in wages to avoid the schooling effort. On the other side, individuals do not find optimal investing in education in order to pay for extending the lifespan, since the lack of medical progress makes ineffective to invest in health care. Therefore, Fig. 5(c) further supports the result that medical progress is crucial for explaining the increase in educational attainment. Moreover, we find that increasing wages without medical progress that extends the life span tends to reduce the educational attainment.

4.2 Retirement age

Figure 6 shows the evolution of the optimal age of retirement across birth cohorts in the benchmark and in the two counterfactuals. We make use of box-plots to simultaneously show the median values and the dispersion. Figures 6(a) and 6(b) correspond to the benchmark model. In Figure 6(a) we can see the evolution of the retirement age for the whole sample of individuals simulated. In Figure 6(b) we show the evolution of the retirement age conditional on the educational attainment. Figures 6(c)-6(d) depict the optimal retirement ages by educational attainment under the two counterfactual experiments. Our two counterfactuals allow us to understand the different effects driving the evolution in the retirement age.

We can see in Fig. 6(a) that the average retirement age across cohorts first decreases and then it increases sharply. This is explained by four different effects. First, following d'Albis et al. (2012) and Sánchez-Romero et al. (2016), mortality improvements produce a differential effect on retirement due to the *lifetime human wealth* effect and the *years-to-consume* effect. Lifetime human wealth effect stands for the positive impact that a mortality decline has on consumption, due to the fact that it raises the likelihood of receiving a future labor income stream. In this model, however, we have cancelled out the lifetime human wealth effect by not allowing individuals to purchase annuities and also by not imposing a borrowing constraint. The years-to-consume effect reflects the overall reduction in consumption due to a longer lifespan. Although these two effects directly affect on consumption, given that consumption and leisure —i.e., retirement— are both normal goods, the impact of mortality on consumption is translated into retirement as it is shown in Eq. (**??**). As a consequence, the increase in the lifespan, due to medical progress, leads individuals to retire later (i.e., years-to-consume effect). This is clearly seen

Julia/RetbyWageEduTotal.jpeg

(a) Benchmark (total)

Julia/RetbyWageEduB.jpeg

(b) Benchmark

Julia/RetbyWageEduB01.jpeg

(c) Constant prices

Julia/RetbyWageEduB02.jpeg

(d) No medical progress

Figure 6: Retirement age by educational attainment.
Source: Authors' simulations.

in Fig. 6(c) in which there is medical progress and prices are kept constant.[12] On the contrary, the initial decline in the retirement age is explained by an income effect caused by the increase in wages, which is used to buy more

[12]Allowing individuals to purchase annuities will trigger the lifetime human wealth effect. Hence, the decline in retirement will be more pronounced, ceteris paribus the maximum longevity.

leisure time. In this regard, Fig. 6(d) shows that when medical progress do not allow further increases in lifespan, the rise in wages lead to early retirement. However, comparing the cohort born in 1910 in Fig. 6(c) to that in Fig. 6(d), we also find the technological progress increases the initial retirement age. The intuition is simple. The increase in productivity raises wages and increases the benefits of continue working. Last, the sharp increase in the retirement age from cohort 1940 onwards is also driven by a compositional effect. As it is shown in Fig. 5(a), the share of individuals attaining postsecondary education increases with later-born cohorts.

Two additional facts are worth noticing. The first fact is the progressive increase across cohorts in the dispersion of the retirement age, except for the cohort born in 1970. On the one hand, this is due to the increasing difference across cohorts in the retirement age between educational groups, see Fig. 6(b). On the other, we have that the shares of the educational groups become similar in size, which also increases the variance between groups. The second fact is that the model is capable of replicating the positive relationship between a longer schooling period and the retirement age, while individuals with primary education work more years than those with postsecondary education.

4.3 Health care expenditure

Figure 7 shows the evolution of the life-cycle health care spending share, i.e. the present value of total health care spending over the life-course in relation to the present value of life-cycle income, across birth cohorts in the benchmark and in the two counterfactuals. Again, the use of box-plots illustrates median values and dispersion.

As we see from panels (a) and (b) in Figure 7, the health care spending share increases strongly for successive cohorts. Assuming a mean spending age at around 70, the average spending shares of 10.2 percent, 11.6 percent and 13.5 percent for the birth cohorts 1910, 1920 and 1930 are well in line with the cross-sectional spending shares in the years 1980, 1990 and 2000, respectively. For an ongoing trend to labour productivity and medical progress, and absent any policy intervention, our model then predicts spending shares of around 20 percent in 2020 and 38.5 percent in 2050. These spending trends are not grossly out of line with similar results reported e.g. by Hall and Jones (2007) and Jones (2016b).

Our numerical analysis also shows that the variation in health care spending increases over time both across the whole population as well as both within and across the educational subgroups. Notably, the spending ratio between the post-secondary educated and those with primary education reverses from

Julia/HealthToIncomeBTotal.jpeg

(a) Benchmark

Julia/HealthToIncomeB.jpeg

(b) Benchmark

Julia/HealthToIncome01.jpeg

(c) Constant prices

Julia/HealthToIncome02.jpeg

(d) No medical progress

Figure 7: Total health expenditure to human capital ratio by educational attainment.

Source: Authors' simulations and Hall and Jones (2007) (red diamonds).

0.898 for the 1910 cohort to 1.504 for the 1970 cohort, the switch occurring for the 1930 cohort, which in the cross-section corresponds to around the year 2000. Notably, this is consistent with recent evidence reported by Dickman et al. (2016) on health care spending patterns by income. The increase in variation across educational groups can be explained by the complementarity between income and medical effectiveness as determinants of health care expenditure. To see this, recall from Eq. (19) that health investments h increase in a multiplicative way with the willingness to pay, as measured by VOD and VOL, which we will show to increase with income and education further on below, and with the state of medical technology. If the latter is low, then it is not worth to spend much on health care regardless of income, whereas medical progress magnifies the higher propensity of well-educated high income earners to spend on elective health care. This is consistent with the mechanism identified by Frankovic and Kuhn (2019). Furthermore, emergency care is a more prominent component of health care spending for the early-born cohorts. The higher mortality risk of the lesser-educated cohorts then implies greater total health care spending on their part. The reversal in spending then occurs only once medical progress has raised the spending incentive of the better-educated to a sufficient extent.

Finally, consideration of the counterfactual scenarios depicted in panels (c) and (d) in Figure 7 is instructive in regard to the drivers of health care spending growth. The shutting down of productivity growth, and consequently earnings growth, as in panel (c), strongly curbs the growth in the health care spending share, which is well in line with Hall and Jones (2007), Fonseca et al. (2013) and Frankovic and Kuhn (2018). Interestingly, however, the shutting down of medical progress brings the growth in the health share almost to a standstill: health care spending continues to grow but only in line with income growth. This suggests that medical progress is the more important driver of health care spending, a finding that is well in line with Fonseca et al. (2013) and Frankovic and Kuhn (2018).

4.4 Life expectancy

In this section we use the model to study the increase in life expectancy across cohorts and the causes of the increasing gap in life expectancy by education. Figure 8 shows the evolution of the life expectancy at age 14 across cohorts. Similar to figs. 6 and 7 we divide the plot in four panels. The top panels correspond to the benchmark simulation, whereas the bottom panels depict the results for the counterfactuals.

Fig. 8(a) shows the increase in life expectancy (at age 14) from 54.7 years (1910 cohort) to 64 years (1970 cohort), which is in line with estimates of cohort life expectancy done by the US Social Security Administration (Bell et al., 1992). Fig. 8(b) complements the previous figure by showing that the rise in life expectancy is faster for individuals with postsecondary education (from 55.4 to 64.8 years) than for individuals with primary education (from 54.4 to

Julia/LEe14USRes.jpeg

(a) Benchmark (Total)

Julia/LE14byWageEduB.jpeg

(b) Benchmark

Julia/LE14byWageEduB01.jpeg

(c) Constant prices

Julia/LE14byWageEduB02.jpeg

(d) No medical progress

Figure 8: Life expectancy at age 14 by educational attainment.
Source: Authors' simulations and Bell et al. (1992) (red diamonds).

60.2 years). Thus, the increasing trend in life expectancy across cohorts is not only driven by the increase in life expectancy for each education group, but also by the change in the educational distribution of cohorts from primary to postsecondary education (see Fig. 5).

Looking at Fig. 8(d), we can see that by shutting down medical progress the conditional life expectancy by education increases very mildly across cohorts.

Therefore, medical progress explains completely the increase in life expectancy. If we control for changes in the educational composition by comparing Fig. 8(c) to Fig. 8(b), we find that when there is technological progress the rise in prices, including labor income, accounts between sixteen and twenty percent of the observed increase in life expectancy. Thus, since the sum of the effect of medical progress and that of prices on life expectancy is greater than one, the model reports that the rise in prices reduce the benefits of medical progress on life expectancy.

An important feature of the model is that it allow us to replicate the increasing gap between the minimum life expectancy and the maximum life expectancy (see Fig. 8(a)). In particular, we obtain an increasing gap between the lowest and the highest life expectancies at age 14 from 3 years, for the cohort born in 1910, to more than 10 years, for the cohort born in 1970. The average gap in life expectancy between individuals with postsecondary and with primary education is of 1 year for the 1910 cohort and 4.6 for the 1970 cohort. To explain the causes of the increasing gap, figs 8(b)–8(d) provide the necessary information. Since individuals will devote an increasing share of their income to health care as the income rises (Hall and Jones, 2007), the combination of an increasing income with a more effective medical progress leads to a divergence in health care spending across education groups and hence in life expectancy.

4.5 Value of life (VOL)

We conclude with a brief take on how the VOL at age 14, i.e. at the age at which individuals start to make economic decisions, emerges for successive cohorts over time. Figure 9 presents the now familiar scenarios. Panels (a) and (b), relating to the benchmark, document a strong increase in the VOL for the whole population, amounting to an increase by a factor of 5.5 from the 1910 to the 1970 cohort, as well as for all education groups. Note that this is very well in line with recent evidence by Costa and Kahn (2004). At the same time, there is no clear-cut trend to the variation in the VOL: While the standard deviation is increasing, this is mostly amounting to a scale effect. Strikingly, when comparing the median VOL across the groups with post-secondary as opposed to primary education, inequality is declining: Here, the VOL of the highest educated is 2.461 times the VOL of the lowest educated for the 1910 cohort, but this ratio diminishes to 1.787 for the 1970 cohort. This can be interpreted in two ways: In a straightforward way, the shrinking in the dispersion of the VOL across education groups reflects the disproportionate increase in the health care spending share of the higher educated. Recall here from Eq. 17 that the VOL increases with the level of consumption. Thus, to the extent that the better educated use a greater and increasing share of their income gains for health care spending rather than consumption, this suggests a weaker increase in their VOL. Alternatively, one could interpret the VOL at age 14 as a summary measure of life-cycle utility. Under this interpretation, the discrepancy in the VOL across education groups suggests significant differences in welfare. Surprisingly, however, the increase in inequality with respect to many of the life-cycle indicators does not translate into an increase in inequality if measured by the ratio of VOL across education groups.

Finally, the trend of the VOL over time that is depicted in panels (c) and (d) of Figure 9 reflects in a straightforward way the underlying counterfactuals: In the absence of productivity growth in panel (c), the VOL remains broadly flat at a relatively low level. While the health share increases moderately, consumption can be kept at a constant level despite stagnant productivity growth. This is because the increase in life expectancy translates into greater incentives to acquire better education and a shift of the population into higher education groups. The resulting education-driven increase in earnings then allows individuals to accommodate the greater health care spending without having to compromise much on consumption. This is reflected in the stagnant VOL. In contrast, the absence of medical progress in panel (d) leads to a strong increase in the VOL across the board. The reason is that productivity-driven increases in income are predominantly spent on consumption and leisure. The resulting boost to the VOL reflects a growing unmet demand for longevity. As it turns out, this demand cannot be met, as in the absence of medical progress additional health care spending remains ineffective in raising longevity.

Julia/VOLBTotal.jpeg

(a) Benchmark (Total)

Julia/VOLB.jpeg

(b) Benchmark

Julia/VOLB01.jpeg

(c) Constant prices

Julia/VOLB02.jpeg

(d) No medical progress

Figure 9: Value of life (VOL) by educational attainment.
Source: Authors' simulations and Costa and Kahn (2004) (red diamonds).

5 Conclusion

In this paper we have developed a normative framework to study the increase in within-cohort inequality in wealth and in life expectancy. Following the reports by the National Academies of Sciences, Engineering, and Medicine (2015) and OECD (2017), we have taken into account that the increase in inequality is not only affected by endogenous decisions in labor supply, consumption, and wealth accumulation, but also by compositional effects and selectivity. In order to control for the last two factors —i.e., compositional effects and selection bias— in a lifecycle model with endogenous decisions on schooling, consumption, labor supply, and life expectancy, we have considered individuals who are heterogeneous at three margins: innate ability, initial health deficits, and (effort) cost of schooling.

Using our normative framework we have investigated how medical progress and productivity growth may trigger the observed increase in within-cohort inequality across educational groups. In so doing, we have run counterfactual experiments in which we shut down the influence of each factor in alternate turns in order to isolate its contribution on the increase in within-cohort inequality. We have found that medical progress is the main driver explaining the rise in educational attainment, retirement age, health care spending, and life expectancy. Moreover, technological progress, without medical progress, produces an adverse selection into lower educational attainment and early retirement. Our simulation results also show that the combination of technological progress with medical progress considerably contribute to widening health care spending across educational groups, and hence they can explain the increase in the life expectancy gap between income groups.

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A The economic problem

The current value Hamiltonian during the three different periods are:

Schooling period (for $x_0 \leq t \leq E$)

$$\begin{aligned} H_1 = & S(u(c) - \phi) + \lambda_a(ra - c - p_h h - p_\mu \mu(D)) \\ & + \lambda_D \beta_d (D - Ah^\eta - \gamma_d) - \lambda_S \mu(D) S. \end{aligned}$$

Working period (for $E < t \leq R$)

$$\begin{aligned} H_2 = & S(u(c) - \alpha v(\ell)) + \lambda_a(ra + w\ell - c - p_h h - p_\mu \mu(D)) \\ & + \lambda_D \beta_d (D - Ah^\eta - \gamma_d) - \lambda_S \mu(D) S. \end{aligned}$$

Retirement period (for $R < t \leq T$)

$$\begin{aligned} H_3 = & S(u(c) + \varphi) + \lambda_a(ra - c - p_h h - p_\mu \mu(D)) \\ & + \lambda_D \beta_d (D - Ah^\eta - \gamma_d) - \lambda_S \mu(D) S. \end{aligned}$$

The term $u(c)$ is the utility from consumption, ϕ stands for the disutility of schooling, $v(\ell)$ is the disutility of work, φ is the utility from being retired.

The optimality conditions for this problem are:

- First-order conditions:

$$\frac{\partial H}{\partial c} = 0, \qquad \frac{\partial H}{\partial h} = 0, \qquad \frac{\partial H}{\partial \ell} = 0, \forall s \in (S, R). \tag{28}$$

- Envelope conditions:

$$\dot{\lambda}_a - \rho\lambda_a = -\frac{\partial H}{\partial a}, \quad \dot{\lambda}_D - \rho\lambda_D = -\frac{\partial H}{\partial D}, \quad \dot{\lambda}_S - \rho\lambda_S = -\frac{\partial H}{\partial S}. \tag{29}$$

- Transversality conditions:

$$\lambda_a(T) = 0, \qquad \lambda_S(T) = 0, \qquad \lambda_D(T) = -\nu, \qquad H(T) = 0, \tag{30}$$

where ν is the Kuhn-Tucker multiplier associated to the maximum number of health deficits.

Given E and R we obtain the first-order conditions for c, ℓ, and h:

$$Su'_c(c) = \lambda_a, \tag{31}$$

$$S\alpha v'_\ell(\ell) = w\lambda_a, \tag{32}$$

$$p_h = -\psi^D \beta_d A\eta h^{\eta-1}. \tag{33}$$

The envelope conditions or the dynamics of the adjoint variables are:

$$\dot{\lambda}_a = \lambda_a(\rho - r), \tag{34}$$

$$\dot{\lambda}_D = \lambda_D(\rho - \beta_d) + \lambda_a \mu'(D)(\psi^S + p_\mu), \tag{35}$$

$$\dot{\lambda}_S = \lambda_S(\rho + \mu(D)) - u(c, \ell), \tag{36}$$

where we define $\psi^S = \frac{\lambda_S S}{\lambda_a}$ as the value of life. Also, let us define $\psi^D = \frac{\lambda_D}{\lambda_a}$ as the value of health deficits.

Combining the first-order conditions with the envelope conditions, we get the following dynamics for the control variables

$$\frac{\dot{c}}{c} = \frac{-u''_{cc}(c)}{cu'_c(c)}(r - \rho - \mu(D)), \tag{37}$$

$$\frac{\dot{\ell}}{\ell} = \frac{v'_\ell(\ell)}{\ell v''_{\ell\ell}(\ell)} \left(\frac{\dot{w}}{w} + \rho - r + \mu(D)\right), \tag{38}$$

$$\frac{\dot{h}}{h} = \frac{1}{1-\eta}\left(r - \beta_d + g_h - g - \frac{\mu'(D)\left(\psi^S + p_\mu\right)}{-\psi^D}\right). \tag{39}$$

Integrating the shadow prices with respect to age and using the boundary conditions we obtain the values for each state variable:

Value of money (VOM)

$$\lambda_a(s) = \lambda_a(0)e^{(\rho-r)s}. \tag{40}$$

Value of life (VOL)

$$\psi^S(s) = \int_s^T e^{-r(x-s)} \frac{\mathbf{u}(x)}{\mathbf{u}'_c(c(x))} dx. \tag{41}$$

Value of deficits (VOD)

$$\psi^D(t) \; = \; \psi^D(T)e^{-(r-\beta_d)(T-t)} - \int_t^T e^{-(r-\beta_d)(s-t)} \mu'(D(s))(\psi^S(s) + p_\mu(s))ds. \tag{42}$$

A.1 Optimal retirement age

Proof: Let the individual's problem be

$$\mathcal{J}(0) = \int_0^T e^{-\rho t} S(t)\mathbf{u}(t)dt, \tag{43}$$

subject to the lifetime budget constraint

$$\int_0^T e^{-rt}(c(t) + p_h h(t) + p_\mu \mu(D(t)))dt = \int_E^R e^{-rt} w(t)\ell(t)dt. \tag{44}$$

Differentiating $\mathcal{J}(0)$ w.r.t. R and equating the result to zero gives the optimal retirement age condition

$$\int_0^T e^{-\rho t} S(t) \left(\mathbf{u}'_c(t) \frac{\partial c(t)}{\partial R} - \mathbf{u}'_\ell(t) \frac{\partial \ell(t)}{\partial R} \right) dt + \int_0^T e^{-\rho t} \frac{\partial S(t)}{\partial R} \mathbf{u}(t)dt = e^{-\rho R} S(R) \left(\varphi(R) + \alpha v(\ell(R)) \right). \tag{45}$$

Using the FOCs and the adjoint variable associated to assets in the previous equation gives

$$\lambda_a(0) \int_0^T e^{-rt} \left(\frac{\partial c(t)}{\partial R} - w(t) \frac{\partial \ell(t)}{\partial R} \right) dt + \int_0^T e^{-\rho t} \frac{\partial S(t)}{\partial R} \mathbf{u}(t)dt = e^{-\rho R} S(R) \left(\varphi(R) + \alpha v(\ell(R)) \right) \tag{46}$$

Differentiating the budget constraint w.r.t. R and equating the result to zero we have

$$\int_0^T e^{-rt}\left(\frac{\partial c(t)}{\partial R} - w(t)\frac{\partial \ell(t)}{\partial R}\right)dt + \int_0^T e^{-rt}\left(p_h\frac{\partial h(t)}{\partial R} + p_\mu \mu'(D(t))\frac{\partial D(t)}{\partial R}\right)dt = e^{-rR}w(R)\ell(R). \quad (47)$$

By substituting (47) in (46) we have

$$\lambda_a(0)\left(e^{-rR}w(R)\ell(R) - \int_0^T e^{-rt}\left(p_h\frac{\partial h(t)}{\partial R} + p_\mu\mu'(D(t))\frac{\partial D(t)}{\partial R}\right)dt\right) + \int_0^T e^{-\rho t}\frac{\partial S(t)}{\partial R}\mathbf{u}(t)dt = e^{-\rho R}S(R)\left(\varphi(R) + \alpha v(\ell(R))\right) \Rightarrow$$

$$\Rightarrow \lambda_a(0)e^{-rR}w(R)\ell(R) = e^{-\rho R}S(R)\left(\varphi(R)+\alpha v(\ell(R))\right) - \int_0^T\left(e^{-\rho t}\frac{\partial S(t)}{\partial R}\mathbf{u}(t) - \lambda_a(0)e^{-rt}\left(p_h\frac{\partial h(t)}{\partial R} + p_\mu\mu'(D(t))\frac{\partial D(t)}{\partial R}\right)\right)dt \Rightarrow$$

$$\Rightarrow \lambda_a(0)e^{-rR}w(R)\ell(R) = e^{-\rho R}S(R)\left(\varphi(R)+\alpha v(\ell(R))\right) - \int_0^T e^{-\rho t}\left(\frac{\partial S(t)}{\partial R}\mathbf{u}(t) - \lambda_a(t)\left(p_h\frac{\partial h(t)}{\partial R} + p_\mu\mu'(D(t))\frac{\partial D(t)}{\partial R}\right)\right)dt \quad (48)$$

Taking $\lambda_a(t)$ as a common factor inside the integral and using the first-order condition on consumption gives

$$\lambda_a(0)e^{-rR}w(R)\ell(R) = e^{-\rho R}S(R)\left(\varphi(R)+\alpha v(\ell(R))\right) - \int_0^T e^{-\rho t}\lambda_a(t)\left(\frac{1}{S(t)}\frac{\partial S(t)}{\partial R}\frac{\mathbf{u}(t)}{\mathbf{u}'_c(t)} - p_h\frac{\partial h(t)}{\partial R} - p_\mu\mu'(D(t))\frac{\partial D(t)}{\partial R}\right)dt \Rightarrow$$

$$\Rightarrow \lambda_a(0)e^{-rR}w(R)\ell(R) = e^{-\rho R}S(R)\left(\varphi(R)+\alpha v(\ell(R))\right) + \lambda_a(0)\int_0^T e^{-rt}\left(\frac{\mathbf{u}(t)}{\mathbf{u}'_c(t)}\int_0^t \mu'(D(s))\frac{\partial D(s)}{\partial R}ds + p_h\frac{\partial h(t)}{\partial R} + p_\mu\mu'(D(t))\frac{\partial D(t)}{\partial R}\right)dt \quad (49)$$

Changing the order of integration for the first component of the integral gives

$$\lambda_a(0)e^{-rR}w(R)\ell(R) = e^{-\rho R}S(R)\left(\varphi(R) + \alpha v(\ell(R))\right)$$
$$+\lambda_a(0)\int_0^T e^{-rt}\left(\mu'(D(t))\frac{\partial D(t)}{\partial R}\int_t^T e^{-r(s-t)}\frac{\mathbf{u}(c(s),\ell(s))}{\mathbf{u}_c'(s)}ds + p_h\frac{\partial h(t)}{\partial R} + p_\mu\mu'(D(t))\frac{\partial D(t)}{\partial R}\right)dt \tag{50}$$

Using the definition of the value of life and taking $\mu'(D(t))\frac{\partial D(t)}{\partial R}$ as common factor gives

$$\begin{aligned}\lambda_a(0)e^{-rR}w(R)\ell(R) &= e^{-\rho R}S(R)\left(\varphi(R) + \alpha v(\ell(R))\right)\\ &+ \lambda_a(0)\int_0^T e^{-rt}\left(\frac{\partial D(t)}{\partial R}\mu'(D(t))\left(\psi^S(t) + p_\mu\right) + p_h\frac{\partial h(t)}{\partial R}\right)dt\end{aligned} \tag{51}$$

If we solve $\frac{\partial D(t)}{\partial R}$ and change the order of the integrals for the solution of $\frac{\partial D(t)}{\partial R}$, the whole third term cancels out. Then,

$$\begin{gathered}\lambda_a(0)e^{-rR}w(R)\ell(R) = e^{-\rho R}S(R)\left(\varphi(R) + \alpha v(\ell(R))\right) \Rightarrow\\ \Rightarrow \lambda_a(R)w(R)\ell(R) = S(R)\left(\varphi(R) + \alpha v(\ell(R))\right) \Rightarrow\\ \Rightarrow \lambda_a(R)w(R)\ell(R) = S(R)\left(\varphi(R) + \alpha v'(\ell(R))\frac{v(\ell(R))}{v'(\ell(R))\ell(R)}\ell(R)\right) \Rightarrow\\ \Rightarrow \lambda_a(R)w(R)\ell(R)\left(1 - \frac{v(\ell(R))}{v'(\ell(R))\ell(R)}\right) = S(R)\varphi(R).\end{gathered} \tag{52}$$

Using the first-order condition and the instantaneous utility function defined in (11), we get

$$\mathbf{u}_c'(R)w(R)\ell(R) = (1+\sigma_l)\varphi(R). \tag{53}$$

Finally, substituting $\ell(R) = \left(\frac{\mathbf{u}_c'(R)w(R)}{\alpha_l}\right)^{\sigma_l}$ in (53) gives the condition for optimal retirement

$$\mathbf{u}_c'(R)w(R)) = \left((\alpha_l)^{\sigma_l}(1+\sigma_l)\varphi(R)\right)^{\frac{1}{1+\sigma_l}}. \tag{54}$$

B Additional figures and tables

B.1 Length of schooling

Table 4: Average values of initial endowments by educational attainment: Constant prices

	1910	1920	1930	1940	1950	1960	1970
Primary							
$\mathbf{E}[\theta]$	0.116	0.114	0.112	0.110	0.107	0.104	0.102
$\mathbf{E}[D_0]$	0.023719	0.023719	0.023721	0.023720	0.023721	0.023725	0.023747
$\mathbf{E}[\phi]$	0.280	0.281	0.280	0.280	0.279	0.279	0.286
Secondary							
$\mathbf{E}[\theta]$	0.147	0.144	0.143	0.141	0.137	0.129	0.116
$\mathbf{E}[D_0]$	0.023714	0.023712	0.023707	0.023718	0.023706	0.023723	0.023725
$\mathbf{E}[\phi]$	0.262	0.259	0.267	0.274	0.276	0.276	0.275
Postsecondary							
$\mathbf{E}[\theta]$	0.155	0.156	0.154	0.152	0.148	0.146	0.140
$\mathbf{E}[D_0]$	0.023645	0.023658	0.023668	0.023687	0.023676	0.023690	0.023697
$\mathbf{E}[\phi]$	0.226	0.230	0.234	0.239	0.247	0.257	0.268

Table 5: Average values of initial endowments by educational attainment: No Medical Progress

	1910	1920	1930	1940	1950	1960	1970
Primary							
$\mathbf{E}[\theta]$	0.111	0.114	0.118	0.119	0.121	0.122	0.123
$\mathbf{E}[D_0]$	0.023713	0.023718	0.023717	0.023719	0.023714	0.023713	0.023718
$\mathbf{E}[\phi]$	0.282	0.281	0.280	0.277	0.277	0.275	0.274
Secondary							
$\mathbf{E}[\theta]$	0.141	0.144	0.146	0.148	0.151	0.153	0.154
$\mathbf{E}[D_0]$	0.023707	0.023713	0.023703	0.023702	0.023696	0.023711	0.023704
$\mathbf{E}[\phi]$	0.266	0.256	0.251	0.246	0.241	0.236	0.229
Postsecondary							
$\mathbf{E}[\theta]$	0.154	0.156	0.158	-	-	-	-
$\mathbf{E}[D_0]$	0.023686	0.023630	0.023559	-	-	-	-
$\mathbf{E}[\phi]$	0.230	0.226	0.222	-	-	-	-

B.2 Retirement age

Table 6: Retirement

	Birth cohort						
	1910	1920	1930	1940	1950	1960	1970
Benchmark							
Total							
Average	63.49	63.02	62.65	62.47	62.62	63.31	65.15
Std. Dev.	0.80	0.82	0.82	0.96	1.22	1.56	1.63
Primary							
Average	62.73	62.29	61.93	61.59	61.34	61.29	61.44
Std. Dev.	0.10	0.08	0.05	0.03	0.05	0.09	0.17
Secondary							
Average	64.06	63.67	63.31	63.07	63.00	63.17	63.63
Std. Dev.	0.14	0.14	0.10	0.11	0.14	0.19	0.27
Postsecondary							
Average	65.08	64.65	64.35	64.20	64.30	64.78	65.90
Std. Dev.	0.14	0.10	0.09	0.09	0.16	0.29	0.61
Constant prices	1910	1920	1930	1940	1950	1960	1970
Total							
Average	59.70	59.81	59.98	60.16	60.55	61.22	62.28
Std. Dev.	0.86	0.93	1.02	1.14	1.31	1.54	1.69
Primary							
Average	59.11	59.13	59.17	59.20	59.31	59.45	59.72
Std. Dev.	0.05	0.05	0.04	0.02	0.03	0.06	0.09
Secondary							
Average	60.88	60.92	60.99	61.10	61.25	61.50	61.91
Std. Dev.	0.09	0.09	0.08	0.08	0.09	0.11	0.17
Postsecondary							
Average	62.24	62.26	62.36	62.48	62.71	63.07	63.69
Std. Dev.	0.05	0.06	0.05	0.07	0.09	0.15	0.26
No medical progress	1910	1920	1930	1940	1950	1960	1970
Total							
Average	63.34	62.72	62.09	61.51	60.96	60.43	59.95
Std. Dev.	0.73	0.65	0.58	0.48	0.41	0.31	0.22
Primary							
Average	62.70	62.24	61.75	61.29	60.80	60.34	59.91
Std. Dev.	0.12	0.09	0.10	0.08	0.08	0.06	0.07
Secondary							
Average	64.01	63.52	63.02	62.49	62.00	61.50	61.00
Std. Dev.	0.15	0.16	0.14	0.12	0.11	0.08	0.08
Postsecondary							
Average	64.98	64.37	63.80				
Std. Dev.	0.15	0.10	-				

B.3 Health care expenditure

Table 7: Total health care expenditure

	Birth cohort						
	1910	1920	1930	1940	1950	1960	1970
Benchmark							
Total							
Average	0.102	0.116	0.135	0.161	0.202	0.268	0.385
Std. Dev.	0.006	0.005	0.006	0.012	0.024	0.044	0.062
Primary							
Average	0.108	0.119	0.134	0.153	0.181	0.218	0.270
Std. Dev.	0.001	0.002	0.003	0.004	0.007	0.011	0.015
Secondary							
Average	0.097	0.112	0.133	0.160	0.197	0.246	0.311
Std. Dev.	0.002	0.004	0.005	0.007	0.011	0.015	0.021
Postsecondary							
Average	0.097	0.116	0.143	0.180	0.231	0.304	0.406
Std. Dev.	0.003	0.005	0.007	0.011	0.016	0.026	0.045
Constant prices	1910	1920	1930	1940	1950	1960	1970
Total							
Average	0.058	0.063	0.071	0.082	0.099	0.127	0.178
Std. Dev.	0.001	0.003	0.005	0.008	0.014	0.025	0.043
Primary							
Average	0.058	0.063	0.069	0.077	0.088	0.104	0.129
Std. Dev.	0.001	0.001	0.002	0.003	0.005	0.007	0.011
Secondary							
Average	0.058	0.064	0.074	0.086	0.102	0.123	0.156
Std. Dev.	0.002	0.003	0.004	0.005	0.007	0.010	0.015
Postsecondary							
Average	0.061	0.072	0.083	0.099	0.122	0.156	0.210
Std. Dev.	0.003	0.003	0.005	0.007	0.010	0.016	0.029
No medical progress	1910	1920	1930	1940	1950	1960	1970
Total							
Average	0.094	0.099	0.105	0.110	0.115	0.120	0.126
Std. Dev.	0.008	0.007	0.006	0.005	0.004	0.003	0.002
Primary							
Average	0.101	0.105	0.109	0.113	0.117	0.121	0.126
Std. Dev.	0.000	0.001	0.001	0.001	0.001	0.001	0.001
Secondary							
Average	0.086	0.090	0.094	0.099	0.104	0.109	0.115
Std. Dev.	0.002	0.002	0.002	0.002	0.002	0.003	0.003
Postsecondary							
Average	0.082	0.088	0.095				
Std. Dev.	0.002	0.002	-				

B.4 Life expectancy at age 14

Table 8: Life expectancy at age 14

	Birth cohort						
	1910	1920	1930	1940	1950	1960	1970
Benchmark							
Total							
Average	54.66	55.21	55.94	56.92	58.37	60.50	64.01
Std. Dev.	0.87	0.92	1.02	1.17	1.45	1.93	2.41
Primary							
Average	54.37	54.89	55.51	56.31	57.40	58.69	60.22
Std. Dev.	0.80	0.82	0.86	0.89	1.00	1.09	1.09
Secondary							
Average	54.87	55.45	56.27	57.23	58.42	59.92	61.74
Std. Dev.	0.81	0.84	0.91	0.94	1.03	1.13	1.26
Postsecondary							
Average	55.37	56.17	57.26	58.40	59.92	62.00	64.85
Std. Dev.	0.83	0.85	0.91	1.01	1.10	1.33	1.86
Constant prices	1910	1920	1930	1940	1950	1960	1970
Total							
Average	54.36	54.81	55.45	56.24	57.30	58.83	61.17
Std. Dev.	0.82	0.85	0.94	1.03	1.22	1.51	2.03
Primary							
Average	54.21	54.61	55.17	55.86	56.70	57.80	59.21
Std. Dev.	0.78	0.78	0.85	0.90	0.97	1.05	1.19
Secondary							
Average	54.65	55.11	55.79	56.57	57.53	58.72	60.37
Std. Dev.	0.81	0.84	0.88	0.91	1.02	1.09	1.27
Postsecondary							
Average	55.09	55.99	56.39	57.36	58.58	60.14	62.45
Std. Dev.	0.89	0.68	0.85	1.00	1.08	1.30	1.62
No medical progress	1910	1920	1930	1940	1950	1960	1970
Total							
Average	53.99	53.98	54.00	53.98	54.02	54.03	54.03
Std. Dev.	0.79	0.79	0.78	0.77	0.77	0.76	0.76
Primary							
Average	53.80	53.84	53.89	53.89	53.97	54.00	54.01
Std. Dev.	0.76	0.75	0.75	0.74	0.76	0.75	0.75
Secondary							
Average	54.18	54.19	54.29	54.37	54.40	54.45	54.50
Std. Dev.	0.75	0.79	0.77	0.79	0.75	0.80	0.76
Postsecondary							
Average	54.58	55.09	55.64				
Std. Dev.	0.81	0.60	-				

B.5 Value of life

Table 9: Value of life (VOL)

	Birth cohort						
	1910	1920	1930	1940	1950	1960	1970
Benchmark							
Total							
Average	684111	892243	1168346	1567871	2155965	2913851	3815051
Std. Dev.	232519	305225	391427	528922	701790	839938	717559
Primary							
Average	462619	619126	822243	1083929	1410520	1813553	2286107
Std. Dev.	4418	5355	6739	8555	8306	5222	8030
Secondary							
Average	763651	1018497	1339402	1724386	2166363	2596531	3017395
Std. Dev.	49919	62712	78478	105681	126750	135467	124493
Postsecondary							
Average	1138689	1494564	1931711	2469912	3055004	3641267	4084548
Std. Dev.	71421	85618	119716	153777	217030	316358	440597
Constant prices	1910	1920	1930	1940	1950	1960	1970
Total							
Average	268381	274404	282436	289274	303833	325421	343656
Std. Dev.	63343	68830	74988	79522	87691	93901	87264
Primary							
Average	225592	225817	226353	226877	227173	227525	227107
Std. Dev.	2830	2771	2879	2932	2959	2880	2520
Secondary							
Average	352408	350028	347302	342146	335044	321236	300155
Std. Dev.	16222	16857	17008	17852	17457	15983	11813
Postsecondary							
Average	492493	490214	483399	476518	462955	444979	417387
Std. Dev.	10018	14719	13648	18548	23347	28920	36612
No medical progress	1910	1920	1930	1940	1950	1960	1970
Total							
Average	625588	790397	990590	1251786	1595158	2020111	2580851
Std. Dev.	195998	228585	266285	312405	357722	368830	350419
Primary							
Average	462974	621279	829798	1101080	1456783	1916333	2512423
Std. Dev.	4382	5264	7191	8667	11740	14438	18810
Secondary							
Average	785627	1062968	1422298	1892125	2512870	3323649	4365885
Std. Dev.	42627	52266	62561	72320	78599	83489	83838
Postsecondary							
Average	1191093	1602757	2150000				
Std. Dev.	40268	27160	-				

B.6 Lifetime income (HK)

Table 10: Lifetime income (HK)

	Birth cohort						
	1910	1920	1930	1940	1950	1960	1970
Benchmark							
Total							
Average	189596	225931	272482	338671	438644	583729	819274
Std. Dev.	45004	55129	67039	87813	118076	153876	162162
Primary							
Average	145330	175344	211990	257105	312320	381891	470454
Std. Dev.	195	263	448	715	1263	2270	3851
Secondary							
Average	207444	251485	304569	367440	441784	522768	621188
Std. Dev.	9303	11067	13143	17255	20547	21425	18749
Postsecondary							
Average	274325	331234	399861	486339	589721	718136	882207
Std. Dev.	11562	13291	17944	23053	33709	54100	93798
Constant prices	1910	1920	1930	1940	1950	1960	1970
Total							
Average	100312	102106	104538	106913	111875	120085	130381
Std. Dev.	16841	18268	20055	21614	24491	27685	28347
Primary							
Average	88836	89007	89255	89578	89980	90632	91581
Std. Dev.	179	189	205	235	287	383	511
Secondary							
Average	122974	122842	122739	122313	121765	119976	116965
Std. Dev.	3675	3866	4029	4435	4533	4499	3561
Postsecondary							
Average	155743	155770	155864	155975	155348	155081	154525
Std. Dev.	2352	3073	2855	4157	5549	7297	10251
No medical progress	1910	1920	1930	1940	1950	1960	1970
Total							
Average	177360	205080	236515	275286	323100	378730	447699
Std. Dev.	38285	41181	44023	47185	49508	46963	41149
Primary							
Average	144919	174335	209870	252493	303928	365503	439653
Std. Dev.	200	259	359	432	561	709	921
Secondary							
Average	210170	254973	308078	372131	450250	544867	657606
Std. Dev.	7577	8313	9090	9743	9624	9589	8561
Postsecondary							
Average	280790	339978	411586				
Std. Dev.	6427	3854	DIV/0!				

Julia/HKToBirthCohortB.jpeg

(a) Benchmark

Julia/HKToBirthCohortB01.jpeg

(b) Constant prices

Julia/HKToBirthCohortB02.jpeg

(c) No medical progress

Figure 10: Present value of lifetime income (HK) by educational attainment