The cross-sectional inequality in lifespan (CAL†): A measure of lifespan variation that reflects mortality history of cohorts

Marília R. Nepomuceno[*1], Qi Cui[2], Alyson van Raalte[1], José Manuel Aburto[1,3] and Vladimir Canudas-Romo[2]

[1] *Max Planck Institute for Demographic Research, Rostock, Germany*

[2] *School of Demography, Australian National University, Canberra, Australia*

[3] *Interdisciplinary Center on Population Dynamics, University of Southern Denmark, Odense, Denmark*

Abstract

Lifespan variation is a key metric of mortality, describing both the individual uncertainty in length of life and the heterogeneity in population health. To date, most empirical studies of long-term trends in lifespan variation have been conducted from period life tables, a temporal dimension which is difficult to interpret at the individual-level. We introduce a new dimension of lifespan variation, named Cross-sectional Inequality in Lifespan(CAL†). Opposed to one-period or one-cohort measures, CAL† combines the mortality experience of several cohorts in a cross-sectional approach. We further demonstrate how to decompose by age and cohort the gap in CAL† between populations,showing specific cohorts that accounted for most of the differences. Higher uncertainty in the timing of death is revealed by CAL† than by lifespan variation measure derived from a period life table-based index.CAL† is a novel and timely lifespan variation measure that provides an alternative insight into the analysis of lifespan variation.

[*] Corresponding author. E-mail: nepomuceno@demogr.mpg.de

Introduction

Population health has traditionally been judged by mean levels including life expectancies and age-standardized death rates. Increasing attention is being paid to the variability in age at death, also known as lifespan variation. At the population level, lifespan variation quantifies the heterogeneity of survival in the population, while at the individual level it is a metric of lifetime uncertainty. If monitoring life expectancy indicates the average progress in increasing longevity, lifespan variation highlights the equality of mortality decline across individuals of different ages (van Raalte et al. 2018).

To date, most empirical studies of long-term trends in lifespan variation have been conducted from period life tables (Alvarez et al. 2019; Colchero et al. 2016; Shkolnikov et al. 2003; Smits and Monden 2009; Vaupel et al. 2011; Vaupel and Canudas-Romo 2003). Trends in cohort lifespan variation have rarely been examined. In large part this is because the most recent completed cohorts were born at a time when infant mortality was substantial. As a result, lifespan variation for these cohorts continues to far surpass the most recently observed period levels (Engelman et al. 2010; Wilmoth and Horiuchi 1999), and the utility of monitoring the lifespan variation of extinct cohorts for macro-level policy decisions or for empowering micro-level life course decisions is likely to be lower.

A key unanswered question is the extent to which individuals internalize their surrounding mortality conditions, and how this information is used in decision-making processes. Imagine two populations with identical period age-specific mortality, but one which experienced rapid mortality decline in the past 50 years, and the other which experienced only moderate decline over a longer time span. Individuals from the two populations would have been exposed to different levels of mortality over their lifetime. This opens intriguing questions on whether people base their own subjective survival expectations (i.e. the most likely age at death) and survival uncertainty (i.e. the uncertainty surrounding that age) on current mortality rates, in line with changing period-based indicators; whether they are more influenced by the health and survival trajectories of their contemporaries from similar birth cohorts, such as school friends, spouses, and siblings; or whether they consider the history or a broader mixture of family members, colleagues and neighbors who may have died some years in the past, and are of mixed birth cohorts. If the latter, this would argue for an approach that mixes periods and cohorts.

For this reason, we introduce a new measure of lifetime uncertainty to the demographic literature, based on the historical mortality experience of all birth cohorts present in a given time. The measure builds on the Cross-sectional Average Length of Life (CAL) approach (Brouard 1986). CAL can be interpreted as the mean length of life lived by an average cohort present in a given time, in terms of the population's mortality experience (Guillot 2003; Riffe and Brouard 2018). Since CAL averages over the past survival of all cohorts present in a given time, it is an informative measure when mortality conditions are changing. The same logic that led to the development of CAL as an indicator of average length of life of cohorts can be used to assess the variability in length of life. To achieve this task, we develop $CAL^{\dagger}$, a new lifespan variation measure, with similar mathematical properties to the life disparity ($e^{\dagger}$) measure (Vaupel and Canudas-Romo 2003). $CAL^{\dagger}$ can be interpreted as the variation in age at death of an average cohort present in a given time, by combining the cohort's mortality experiences in a cross-sectional approach.

In what follows, we discuss the interpretation of lifespan variation at the individual level and its difference in cohort and period perspectives. We present empirical data used in the illustration of $CAL^{\dagger}$ measure and its formulation. We examine the evolution of $CAL^{\dagger}$ in populations with long mortality series and compare these trends with the developments of period and cohort $e^{\dagger}$. We contrast the development of the negative correlation of $CAL^{\dagger}$ and CAL with those of $e^{\dagger}$ and life expectancy in period and cohort perspectives. Finally, we demonstrate how to decompose $CAL^{\dagger}$ by age and cohort and provide R-code for all empirical calculations.

Background

Interpreting lifespan variation at the individual level

As mentioned, one interpretation of lifespan variation is as a metric of individual-level uncertainty in the timing of death. This interpretation sees mortality as a stochastic process resulting from exposure to a set of age-specific death rates, and form the basis for individual's subjective survival expectations and survival uncertainty. Subjective survival probabilities are important because they are instrumental in key life course decisions. Expectations surrounding survival have been argued to impact disparate areas of the life course including savings and

retirement behavior (Hurd et al. 1998; Hurd et al. 2004), the adoption of healthy lifestyles and the propensity to undergo health screening (Picone et al. 2004; Scott-Sheldon et al. 2010), and even the timing of childbearing (Eisenberg and Schenker 1997; Geronimus et al. 1999; Nettle 2010; Rindfuss and Bumpass 1976). Further, in a theoretical framework, Lee and Goldstein (2003) showed that longer survival expectations make experimentation with schooling and career decisions less costly and potentially more rewarding. However, as they acknowledge, gains to life expectancy are not equally spread across age. In high mortality settings, life years are gained mostly over infancy and childhood while in contemporary low mortality settings, such gains accrue mainly in post-retirement ages. This, they argue, partially accounts for why proportional changes in life expectancy do not translate to proportional changes in other life course events.

A key assumption to the Lee and Goldstein (2003) framework, and implicit within much of the literature linking subjective survival expectations with individual behaviors (Hurd, et al. 1998; Hurd et al. 2004; Perozek 2008) is that individuals base their life course decisions on achieving average levels of survival. But equally plausible is that individuals also consider the uncertainty in survival outcomes in planning their lives. In fact, French survey data that elicited subjective survival probabilities of surviving to multiple ages showed that subjective uncertainty about longevity (the standard deviation of their subjective expected age at death), closely matched life table standard deviations in age at death, and had an association with risky behavior that was independent of their subjective life expectancy (Dormont et al. 2018). This is consistent with literature arguing that uncertainty in the timing of death is an important policy consideration and highly undesirable at the individual level (van Raalte et al. 2018).

Given that both objective and subjective lifetime uncertainty are shaped by current and past conditions, an approach that considers mortality conditions from the past to the present can shed light on how individuals' lifetime uncertainty affects their decisions, and complements other measures that consider only current or past mortality.

Measuring lifespan variation – period and cohort perspectives

Several measures have been proposed to analyze lifespan variation such as life disparity (Vaupel and Canudas-Romo 2003), the Gini coefficient (Shkolnikov et al. 2003), the Theil's index (Theil 1967), and standard statistical measures applied to the distribution of age at death (standard

deviation, variance and interquartile range). All of these measures derive from the distribution of ages at death and are highly correlated (Wilmoth and Horiuchi 1999; Kannisto 2000; Anand et al. 2001; Cheung et al. 2005; Vaupel et al. 2011, van Raalte and Caswell 2013). Theoretically, these metrics can be applied to period and cohort approaches, but due to data constraints, and to obtain more timely information, most empirical studies are period-based.

For those lifespan variation metrics derived from one period, the individual uncertainty in the timing of death is based on current age-specific death rates, while a cohort index takes into account the mortality experience of a single cohort from birth until its extinction, reflecting mortality conditions that occurred entirely in the past, and only for a small segment of the total population. The interpretation of these measures depends on the set of age-specific death rates considered by each index. To clarify this point, we turn to the period life expectancies. Demographers usually describe life expectancy as the average lifespan of a hypothetical cohort of individuals who live their lives under current mortality conditions (Preston et al. 2001; Vaupel 2002; Vaupel 2008). However, current mortality rates at each age result not only from the current epidemiological environment, but also from the selective historical mortality experience of the particular cohort that has survived to each age (Vaupel 2002). This makes the interpretation of period expectancies (and variability) somewhat awkward.

CAL, and by extension $CAL^{\dagger}$, on the other hand, measure averages and variability in age at death that unifies both past and present mortality in a cross-sectional approach. As such, the indices can be expected to move more gradually since from one year to the next, the cohorts present in the index will be largely overlapping. Yet compared to pure period or cohort approaches, CAL and $CAL^{\dagger}$ can provide timely information that takes into account not only past mortality conditions but also the current one, drawn from information on the entire population present. Given that period, cohort and CAL-based lifespan variation measures differ with the set of age-specific death rates considered, we expect higher variation in age at death based on $CAL^{\dagger}$ than those from period life tables in populations which mortality has been declining.

Data

We illustrate $CAL^{\dagger}$ to data from Denmark, England and Wales, Finland, France, Italy, the Netherlands, Norway, Scotland, Sweden and Switzerland. We drew data from 1879 to 2013 from the Human Mortality Database (2019). These HMD populations have long enough series of mortality to construct $CAL^{\dagger}$, which justifies our selection. More specifically, we used deaths by Lexis triangles and population counts to calculate cohort age-specific death probabilities, and then, we followed the HMD protocol to construct cohort life tables (Wilmoth 2017). The age- and cohort decomposition of $CAL^{\dagger}$ differences were performed between each country and an average population that represents the average mortality of the 10 selected countries, with each population given the same weight in the average.

Methods

Considering that $CAL^{\dagger}$ and lifespan disparity ($e^{\dagger}$) have similar mathematical properties, in what follows we first describe $e^{\dagger}$. Later, we introduce $CAL^{\dagger}$ and the age- and cohort decomposition of the difference between two $CAL^{\dagger}$.

Life disparity ($e^{\dagger}$)

Life disparity ($e^{\dagger}$) is a lifespan variation measure derived from a life table. At time t, $e^{\dagger}(t)$ represents the average of the product of deaths at each age by their remaining life expectancy. It reflects the life table variation in age at death (Vaupel et al. 2011), as well as the number of years that could be gained if deaths were averted (Vaupel and Canudas-Romo 2003). It is closely related to the entropy of the life table, introduced in demography by Leser in 1955 (Leser 1955), and further explored by several researchers as a measure of death variability (Demetrius 1979; Keyfitz and Golini 1975; Vaupel 1986, Goldman and Lord 1986; Nusselder and Mackenbach 1996; Wilmoth and Horiuchi 1999). In 1977, Keyfitz used the life table entropy to calculate the effect of instantaneous change in mortality in life expectancy (Keyfitz 1977). Decades later, Vaupel and Canudas-Romo (2003) revealed that the life table entropy can be expressed as the ratio given by life disparity and life expectancy at birth, $e^{\dagger}/e_0$. Following this relation and defining the radix of the life table is equal to 1 ($\ell(0,t)=1$), $e^{\dagger}(t)$ can be written as

$$e^{\dagger}(t) = -\int_0^{\omega} \ell(x,t) \ln[\ell(x,t)]\, dx, \qquad (1)$$

where $\ell(x, t)$ is the life table survival function at age x and time t, and ω is the oldest age attained in the population at time t. More details of this equation can be found in the online supplemental material (OSM-1).

Cross-Sectional Inequality in Lifespan ($CAL^{\dagger}$)

Together with life expectancy, indices of lifespan variation such as $e^{\dagger}$, are measures calculated for one period or for one cohort. To further include the mortality history of all cohorts present at a given time, Brouard (1986) developed the Cross-sectional Average Length of Life (CAL). CAL is a summary mortality measure that takes into account the mortality conditions of all cohorts present in a given time t, and it is calculated as

$$\mathrm{CAL}(t) = \int_0^{\omega} \ell_c(x, t\text{-}x)\, dx \qquad (2)$$

where t is the time period at which the measure is calculated, and $\ell_c(x, t\text{-}x)$ is the cohort life table survival function for the cohort born in year $t\text{-}x$. CAL is comparable to life expectancy by describing the length of life of a population as an average of all the cohort specific survivals. A comparable measure of lifespan variation that includes the historical mortality information of all cohorts present at a given time remains to be defined. To fill this gap, we propose an analogous measure to life disparity in Eq.1, defined as

$$\mathrm{CAL}^{\dagger}(t) = -\int_0^{\omega} \ell_c(x, t\text{-}x) \ln[\ell_c(x, t\text{-}x)]\, dx, \qquad (3)$$

where t is the time period at which the measure is calculated at, and $\ell_c(x, t\text{-}x)$ is the cohort life table survival function for the cohort born in year $t\text{-}x$.

To further justify the use of $\mathrm{CAL}^{\dagger}(t)$, defined in Eq.3 as a measure of variation in age at death, we present the following. In the OSM-2, Table A1 shows the mathematical similarity between life expectancy and life disparity, and the two measures including cohort information in $\mathrm{CAL}(t)$ and $\mathrm{CAL}^{\dagger}(t)$. Also, we investigate the relationship between $\mathrm{CAL}^{\dagger}(t)$ and the standard deviation derived from the cohort survival functions $\ell_c(x, t\text{-}x)$, which is strongly correlated (Figure A1 in the OSM-3). Finally, OSM-3 shows in details the calculations of $\mathrm{CAL}^{\dagger}(t)$.

Decomposition of the difference between two CAL†

In addition, we introduce the age and cohort decomposition of the difference between two CAL†. The reason for this is the strong interest, especially among demographers, in disentangling a change or difference in aggregate measures when comparing two populations or one population over time. Decompositions can be performed by any underlying parameter such as age, cause of death, and cohort. Here, our aim is to decompose a gap between two CAL† into its age and cohort contributions. Since we are dealing with cross-sectional measures, traditional age-decomposition methods for life expectancy (Arriaga 1984; Andreev et al. 2002; Vaupel and Canudas-Romo 2003) and variation in age at death (Shkolnikov et al. 2003; Zhang and Vaupel 2009; Shkolnikov et al. 2011; Gillespie et al. 2014, Aburto et al. 2019) have to be extended to include age- and cohort components (Canudas-Romo and Guillot 2015).

To asses differences between two CAL†, we examine the derivatives of CAL† with respect to the variable of interest (*e.g*. time or comparisons between two populations) as,

$$\dot{\text{CAL}}^{\dagger}(t) = -\int_0^{\omega} \ell_c(x, t-x)\,\{\ln[\ell_c(x, t-x)] + 1\} \sum_{i=0}^{x-1} \frac{{}_1\dot{p}_i(t-x)}{{}_1p_i(t-x)}\, dx. \qquad (4)$$

where the notation of a dot on top of a variable denotes the derivative of a function with respect to the variable of interest. We denote ${}_1p_i(t-x)$ as the probability of surviving from age *i* to *i*+1 for the cohort reaching age *x* at time *t*. By expressing cohort survival function as the product of single age probabilities of surviving from age zero to age *x* as $\ell_c(x, t-x) = {}_1p_0(t-x)\ {}_1p_1(t-x) \ldots {}_1p_{x-1}(t-x)$, we can separate the derivative of cohort survivals into age contributions (Canudas-Romo and Guillot 2015). The relative derivatives with respect to the age of the cohort survival probabilities, denoted as $\frac{{}_1\dot{p}_i(t-x)}{{}_1p_i(t-x)}$ in Eq.4, correspond to the contribution of the age *i*, of the cohort aged *x* at time *t* in the overall change in CAL†.

Equation 4 includes a component of change, namely the relative derivatives of a single age cohort probabilities of surviving, and two weighting functions: the component of CAL and the component of CAL† at time *t*, or the cohort survival function and the cohort disparity component respectively (see Table A1 in OSM-2 for more details). Thus, although the age-specific survival

component of change is vital to understand the dynamic of this measure of variability, also, the achieved levels of survival and variability are fundamental in the comparison.

Since we used empirical data by single years and single ages, we discretized Eq.3 and Eq.4 in our illustration of CAL†.

Results

Figure 1 presents trends of period ($e^{\dagger}_{0,p}$) and cohort ($e^{\dagger}_{0,c}$) life disparity, and CAL† by sex. The six panels show a declining trend, although at different levels. Variation in age at death transitions from levels as high as 25 years for period and cohort $e^{\dagger}_{0}$ of the late 19th century to low levels around 11 years for $e^{\dagger}_{0,p}$ and CAL† in the 21st century. The trend of CAL† is smoother than $e^{\dagger}_{0,p}$ and $e^{\dagger}_{0,c}$. Compared to $e^{\dagger}_{0,p}$, CAL† moves more gradually because it is less affected by period fluctuations. For instance, Figure 1 shows two peaks of the period life disparity: one around 1919 and another around 1945, a result of the Spanish flu and the two world wars. These peaks reflect the immediate impact of period mortality shocks on the $e^{\dagger}_{0,p}$. Further, for all the years, CAL† is higher than the period variation in age at death, reflecting the high past levels of mortality taken into account by CAL†. For instance, for French women, CAL† was about 36% and 20% higher than $e^{\dagger}_{0,p}$, respectively in 1989 and 2013. Although the higher levels of variation in age at death revealed by CAL† in both years, there is a declining trend of difference between CAL† and $e^{\dagger}_{0,p}$. In Sweden, CAL† was about 15% and 20% higher than $e^{\dagger}_{0,p}$, respectively for men and women, in 1989, and the difference dropped to 11% in 2013 for both sexes. These findings, besides to reveal how lifetime uncertainty changes with the measure, they suggest a reducing difference between current mortality conditions and current mortality rates, as depicted respectively by CAL† and $e^{\dagger}_{0,p}$.

[Figure 1 about here]

Despite differences in magnitude, all three lifespan variation measures ($e^{\dagger}_{0,p}$, $e^{\dagger}_{0,c}$, and CAL†) declined as longevity increased, as presented in Figure 2. This negative correlation suggests that

lower levels of lifespan variation are consistent with higher levels of longevity. Different than in Figure 1, now the associations of the trends are related to the pace of change. The slope (α) presented in each panel of Figure 2 derived from linear regressions controlled by the number of observations in each pair comparisons. While the pair of the CAL measures resemble the end point of the period ones ($e^{\dagger}_{0,p}$ vs. $e_{0,p}$), their change is closer to that observed among the cohort pair ones ($e^{\dagger}_{0,c}$ vs. $e_{0,c}$). CAL and cohort measures decline at a pace of 0.37-0.38 and 0.25-28 per year for females and males respectively, while the period reductions are faster (0.44 and 0.43, respectively for females and males). Steeper decline in period measures may result of the faster reaction of period measures to changes in mortality when compared with the cohort and cross-sectional indices.

[Figure 2 about here]

Table 1 covers period ($e_{0,p}$ and $e^{\dagger}_{0,p}$) and cross-sectional measures (CAL and CAL†) in 2013, and comparisons of indices of lifespan variation between each country and the average population. Gaps in $e^{\dagger}_{0,p}$ and in CAL† reveal which populations have higher or lower inequality in lifespans. Positive gaps correspond to higher lifespan variation in the index compared to the average population. As expected, the gaps differ according to the lifespan variation measure. For French women, although both gaps suggest higher inequality in lifespans than the average population, the gap in CAL† is more than 5 times higher than the gap in $e^{\dagger}_{0,p}$ in 2013. For some populations, not only the magnitude of the gap differs with the measure, but also its direction, such as the case of Italy and England and Wales. These results reveal that the set of age-specific mortality rates considered by each measure can greatly affect population comparisons.

[Table 1 about here]

To better understand the gap in lifespan variation, Figure 3 shows the age and cohort decomposition of the gap in CAL† between Sweden and France against the average population in 2013. These decompositions allow us to attribute the age and cohort contribution to the observed differences in CAL† for females and males, respectively. Positive values (red hues) indicate contributions to higher CAL† in Sweden/France compared to the average population, while blue

hues indicate contributions to lower $CAL^{\dagger}$ in Sweden and France with respect to the average population.

[Figure 3 about here]

Panels A and B present results for Sweden, where lifespan variation was lower than the average; while panels C and D show France, a country that experienced higher lifespan variation than the average according to $CAL^{\dagger}$ (Table 1). In the comparison between Sweden and the average, cohorts born before 1925 and 1929, respectively for women and men, experienced higher disparity across the life course than their counterparts in the average population, although reversing this trend at very old ages. Opposing this, the cohorts born after the 1930s show lower disparity at most ages during the life course, helping to explain the lower overall lifespan variation compared to the average in 2013. For men, cohorts born from the second quarter of the century, particularly in the 1940s and 1950s, contribute substantially to the Swedish lower variation in age at death when compared to the average.

France experienced higher lifespan variation than the average population measured by $CAL^{\dagger}$. For both sexes, the oldest cohorts - those born in the 1910s and 1920s – contributed to decreasing disparity with respect to the average up to the ages 50 and 60, respectively for women and men; however, they gradually lost this advantage, as shown by the positive (red hues) contributions at older ages in more recent periods. As opposed to the oldest cohorts, most of the French cohorts born from the early-1930s onward contributed to higher disparity at young and middle ages compared to their counterparts in the average population. For men, Figure 3 also shows a greater contribution to higher disparity for those cohorts born during the 1950s that reached ages 50-60 in 2013. These contributions, that together with the higher disparity of the oldest cohorts explain the substantial gap in 2013 between France and the average population.

The four panels of Figure 3 show that at some ages the contribution to the difference between Sweden/France and the average population is zero (white color). Specifically, there is a clear cut, at which the contribution to the difference in $CAL^{\dagger}$ of cohorts born around 1930 for females and 1925 for males is zero from birth to the age reached in 2013. If we focus on France, the darkest blue hues at ages between 20 and 30 around 1945 indicate that this age range contributed to decreasing the gap in $CAL^{\dagger}$ with respect to the average population, in a period when mortality

was higher than the average population due to the second World War. This is a specific property of $CAL^{\dagger}$ that can be disentangled and identified with our framework. However, this is consistent with the known property of life disparity measures ($e^{\dagger}$) that unlike life expectancy, which lower mortality at any age is translated into gains in life expectancy, decreases in lifespan variation depend on the ages at which mortality is lower.

Summary of Results

We introduced $CAL^{\dagger}$ as an indicator of lifespan variation that includes the mortality experience of all cohorts present in a given time. By analyzing long time series of $CAL^{\dagger}$ for 10 populations with high-quality historical mortality data, we measured lifespan variation accounting not only one period or one cohort, but considering the past mortality rates previously experienced by cohorts in a cross-sectional approach. $CAL^{\dagger}$ trends are remarkably similar to those of life disparity from a period and a cohort perspective, albeit with different levels. Lower levels of $CAL^{\dagger}$ is consistent with higher levels of longevity measured by CAL. Differences in variation in the age at death are substantial between cohort, period and cross-sectional measures: $CAL^{\dagger}$ reveals higher uncertainty in the timing of death than the period life table-based index. As a result, the gap in lifespan variation between populations greatly change with the measure. Decomposing the differences in $CAL^{\dagger}$ reveals that decreases in lifespan variation depend on the ages at which mortality is lower.

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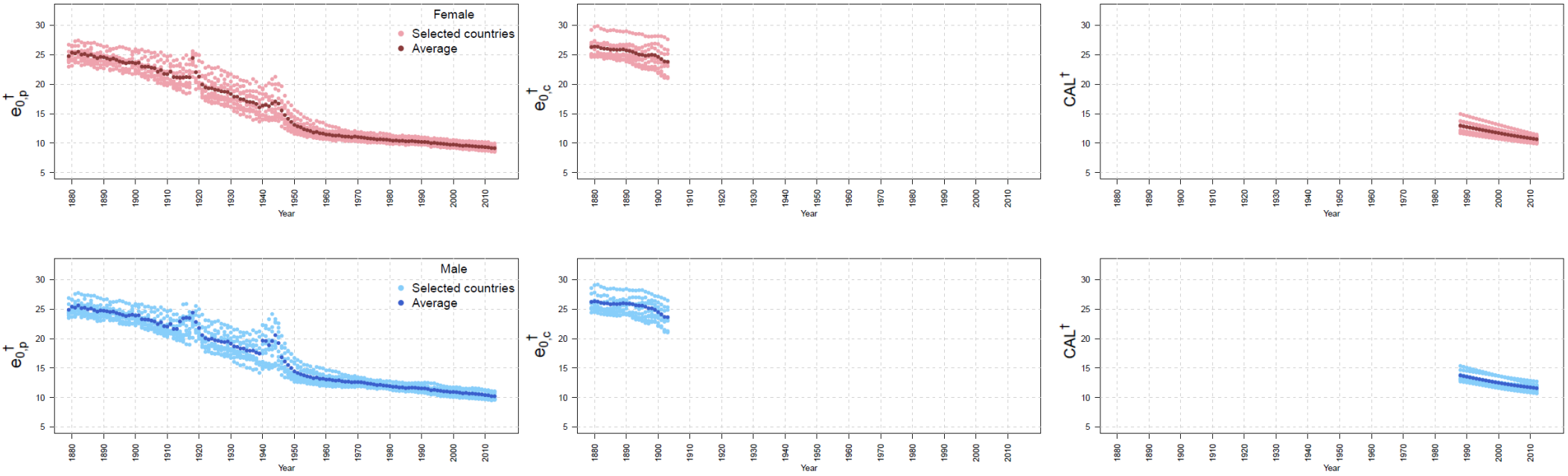
Table 1 – Period life expectancy and life disparity, CAL and CAL†, and gap in lifespan variation measures between each country and the average population in 2013.

Country	$e_{0,p}$	$e^{\dagger}_{0,p}$	Gap in $e^{\dagger}_{0,p}$	CAL	CAL†	Gap in CAL†
Female						
Average	*83.47*	*9.16*	-	*79.95*	*10.67*	-
Denmark	82.31	9.50	0.34	78.60	10.99	0.32
England &Wales	82.96	9.43	0.28	79.64	10.62	-0.05
Finland	83.83	8.86	-0.30	79.90	10.36	-0.31
France	85.05	9.24	0.09	80.80	11.14	0.47
Italy	84.95	8.67	-0.48	79.03	11.45	0.79
Netherlands	83.04	9.18	0.02	80.43	10.35	-0.32
Norway	83.60	9.01	-0.15	81.10	10.15	-0.52
Scotland	81.06	9.96	0.80	77.39	11.12	0.46
Sweden	83.71	8.85	-0.31	81.39	9.91	-0.76
Switzerland	84.75	8.53	-0.63	81.86	10.06	-0.61
Male						
Average	*79.08*	*10.22*	-	*74.54*	*11.60*	-
Denmark	78.26	10.21	-0.01	73.94	11.57	-0.03
England &Wales	79.23	10.32	0.10	75.18	11.21	-0.38
Finland	77.88	10.75	0.53	72.79	11.92	0.33
France	78.77	11.07	0.85	73.50	12.73	1.13
Italy	80.25	9.75	-0.47	73.64	12.16	0.56
Netherlands	79.42	9.68	-0.54	75.57	10.78	-0.82
Norway	79.65	9.82	-0.40	75.78	11.09	-0.51
Scotland	77.03	11.03	0.80	72.56	11.94	0.34
Sweden	80.10	9.61	-0.62	76.69	10.71	-0.89
Switzerland	80.52	9.70	-0.52	76.25	11.43	-0.17

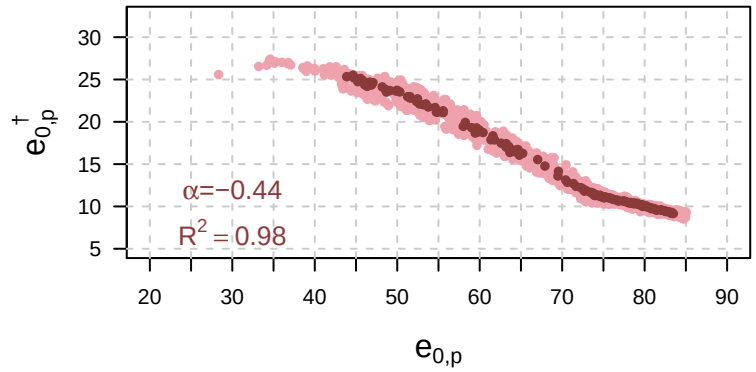
Source: HMD (2019). Calculations by the authors.

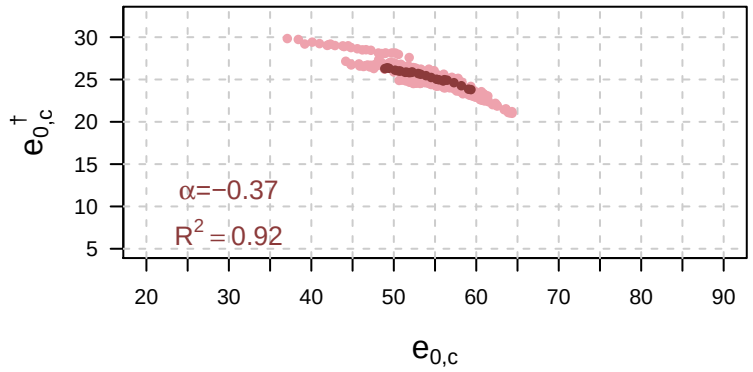
Figure 1 - Period and cohort life disparity and CAL† by sex, 1879-2013.

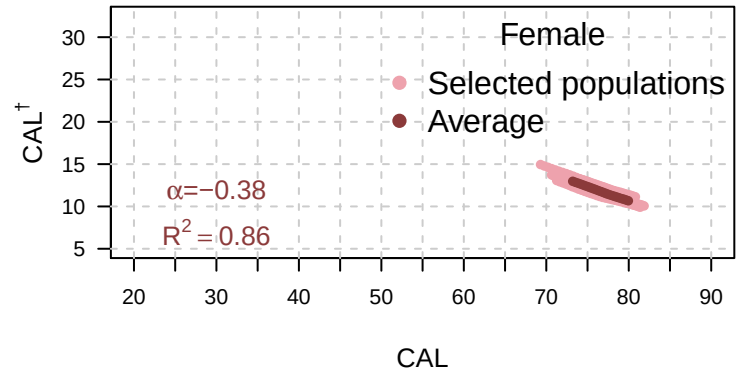


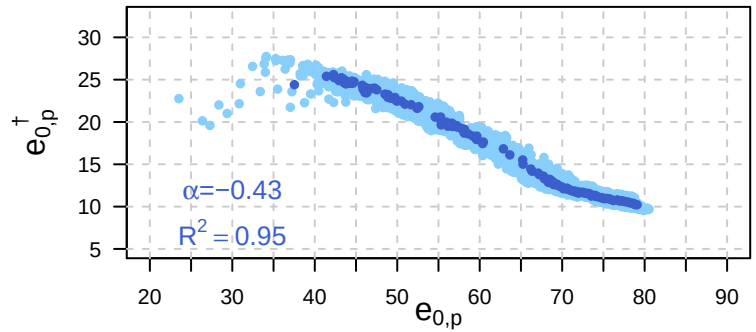
Source: HMD (2019). Calculations by the authors.

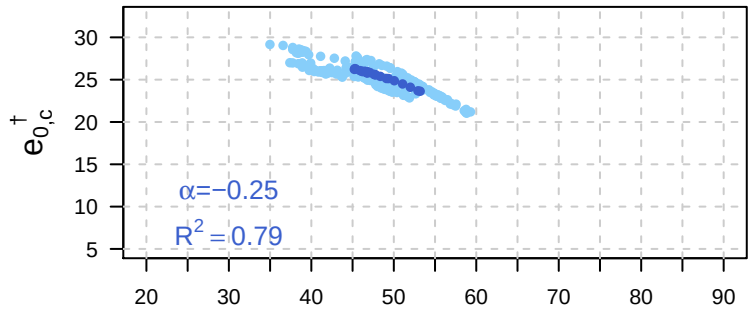
Figure 2 - The relationship between lifespan variation and longevity by sex: $e^{\dagger}_{0,p}$ vs. $e_{0,p}$, $e^{\dagger}_{0,c}$ vs. $e_{0,c}$, and CAL vs. $CAL^{\dagger}$.

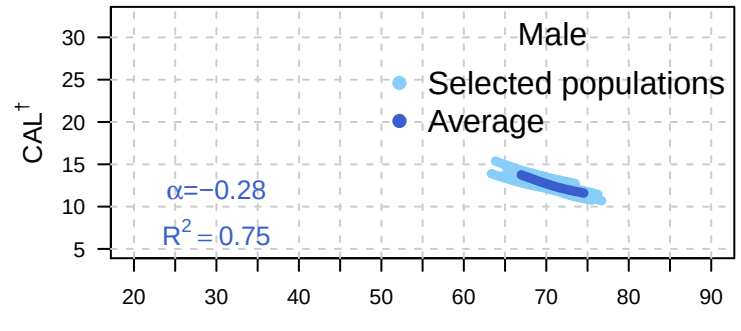






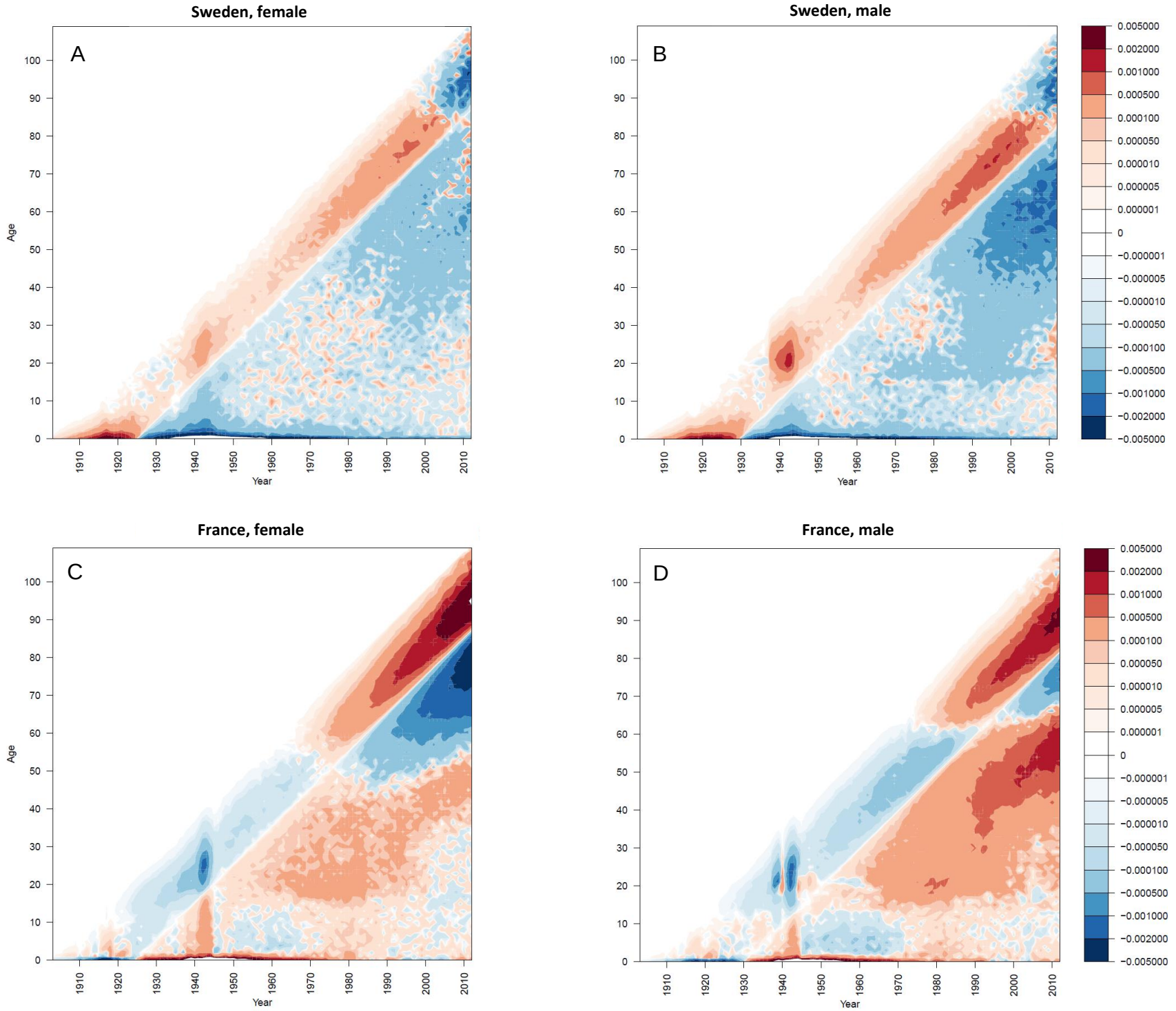






Source: HMD (2019). Calculations by the authors.

Figure 3 - Age and cohort decomposition of the gap in CAL† between Sweden/France and the average population, by sex, 2013.



Source: HMD (2019). Calculations by the authors.

Online Supplementary Material (OSM)

OSM 1.

Measuring variation in age.

Vaupel and Canudas-Romo (2003) first defined the variation in at age at death as the average of the remaining life expectancy weighted by the life table death distribution as

$$e^{\dagger}(t) = \int_0^{\omega} d(x,t)e_0(x,t)dx, \text{(A1)}$$

Where $d(x,t)$ and $e_0(x,t)$ denote correspondingly deaths and remaining life expectancy at age *x* and time *t* and the radix of the population is equal to one, $\ell(0,t) = \int_0^{\omega} d(x,t)dx$. Since the death distribution at age *x* equals the product of survivors by the death rate at this age, and with the definition of remaining life expectancy at age *x* equal to the average survival above age *x*, then equation (A1) can be rewritten as

$$e^{\dagger}(t) = \int_0^{\omega} \mu(x,t) \int_x^{\omega} \ell(a,t)da\,dx, \quad \text{(A2)}$$

where $\mu(x,t)$ is the death rate at age *x* and time *t*. Finally, reversing the limits of integration and recalling the definition of the survival function as $\ell(a,t) = e^{-\int_0^a \mu(x,t)dx}$ returns equation (A2) and the mathematical comparison of the two measures is found in Table A1 in the OSM-2.

OSM 2.

Table A1. Measures of expected years and variation in age at death.

	For one period *t* or one cohort	Including the cohort mortality information of all cohorts present at a given time *t*
Expected years	$e_0(t) = \int_0^{\omega} \ell(x,t)dx$	$\mathrm{CAL}(t) = \int_0^{\omega} \ell_c(x,t-x)dx$
Variation in age at death	$e^{\dagger}(t) = -\int_0^{\omega} \ell(x,t)\ln[\ell(x,t)]\,dx$	$\mathrm{CAL}^{\dagger}(t) = -\int_0^{\omega} \ell_c(x,t-x)\ln[\ell_c(x,t-x)]\,dx$

OSM 3.

Calculation details of $\mathrm{CAL}^{\dagger}(t)$

The standard deviation derived from the cohort survival functions $\ell_c(x, t-x)$, elements also of $\mathrm{CAL}(t)$, were calculated and compared with the values of $\mathrm{CAL}^{\dagger}(t)$ and presented in Figure A1.

[Figure A1 about here]

$\mathrm{CAL}(t)$ and $\mathrm{CAL}^{\dagger}(t)$ as defined in Table A1 are in continuous as well as life expectancy and $e^{\dagger}$, however mortality information is found only in discrete intervals (a year, decade, etc.). To derive the information for the two cohort measures and test the sensitivity of these procedures two different ways of calculating them were followed: i) based on cohort survival functions up to a full age attained by the year of interest *t*, e.g. 0, 1, 2, 3, etc; ii) based on surviving an extra triangle of life as indicated by the dark blue triangles in the lexis diagram below

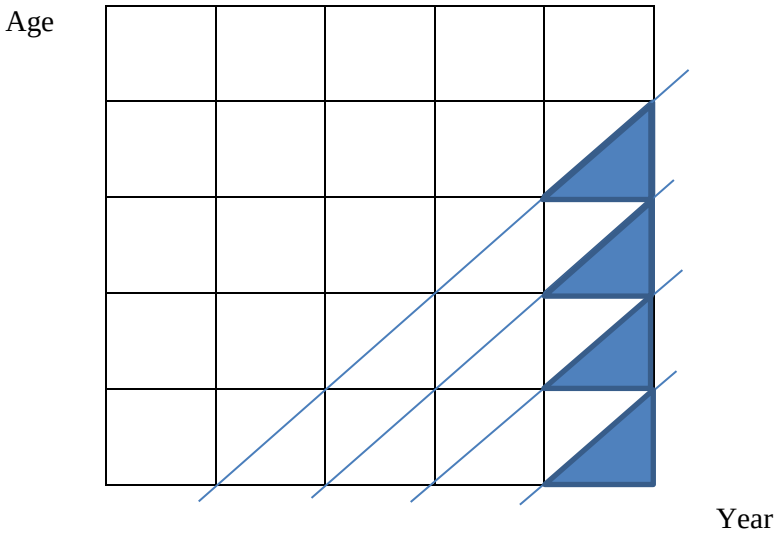


Figure A2. Lexis Diagram of the data selection for calculating $\mathrm{CAL}(t)$ and $\mathrm{CAL}^{\dagger}(t)$

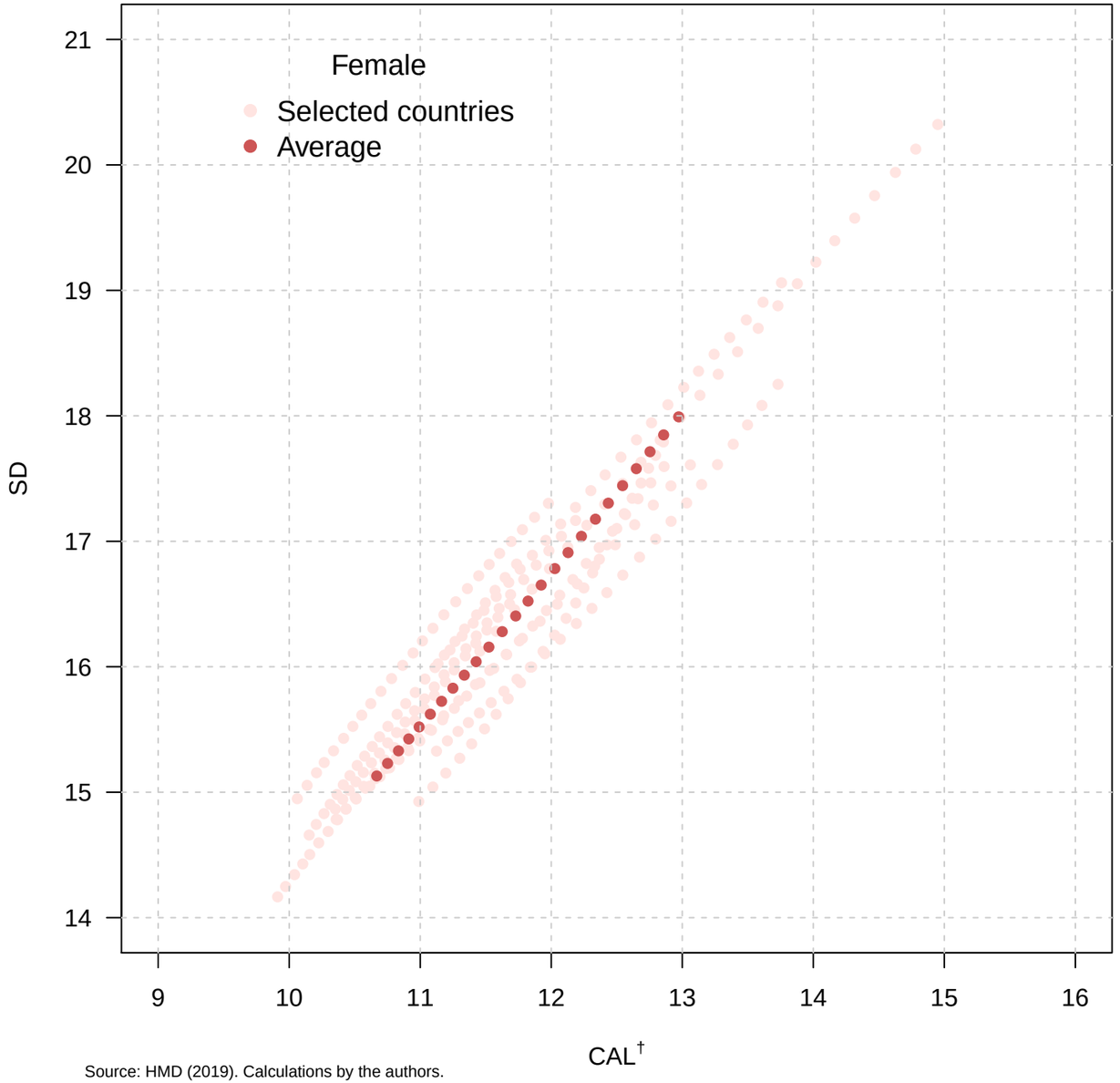
Figure A3 includes the comparison of $\mathrm{CAL}(t)$ and $\mathrm{CAL}^{\dagger}(t)$ measures including or not the triangles as indicated above in (i) and (ii), as well as in the Lexis diagram of Figure A2. The disparity between the two never passed half a year and thus the procedure of only having full

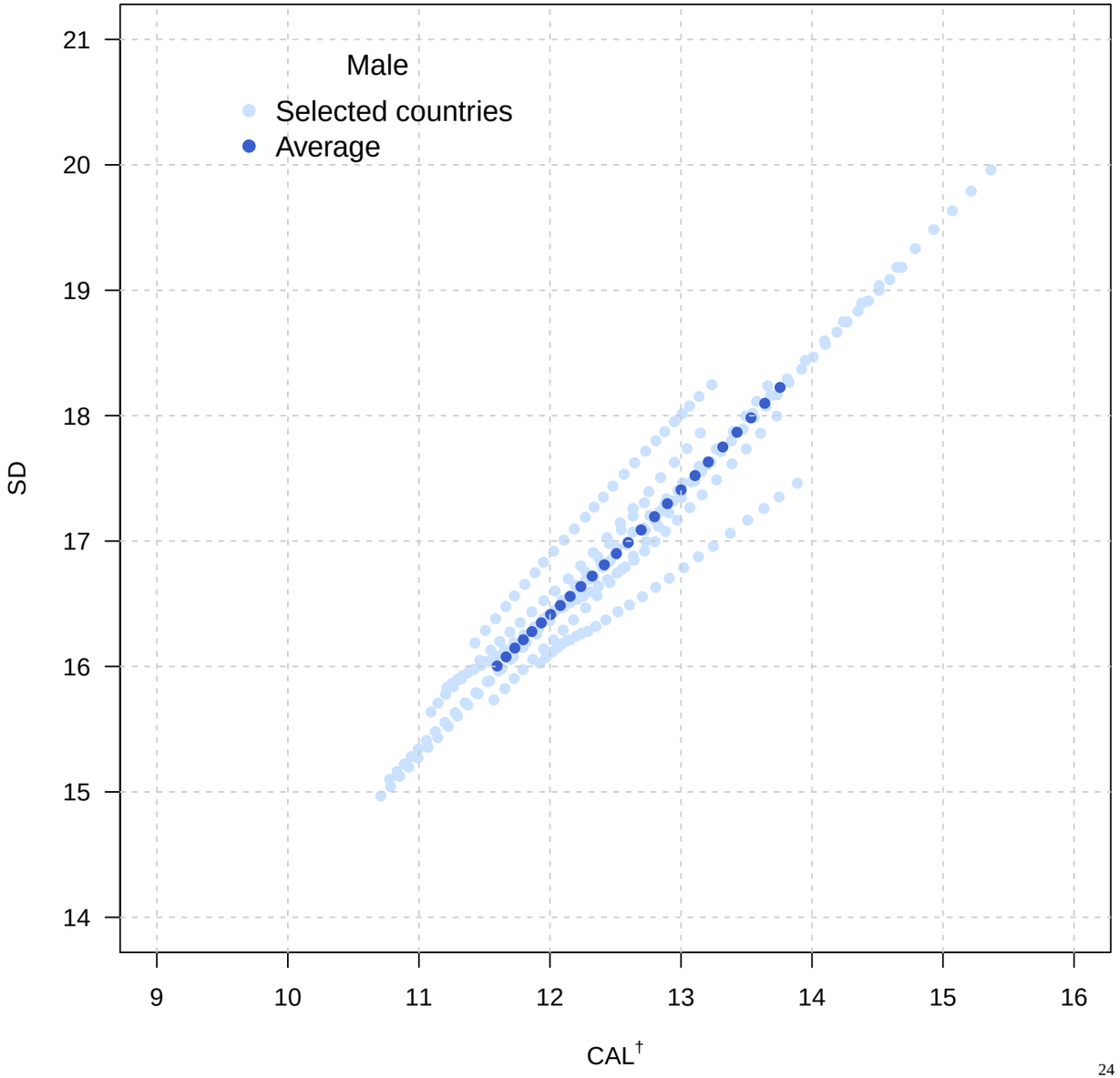
ages was preferred to correspond better with the usual life expectancy and other life table calculations.

[Figure A3 about here]

Finally, calculations for deriving the two cohort measures based on probabilities of surviving and cohort death rates returned identical results (not shown).

Figure A1: Relationship between CAL† and the standard deviation derived from the cohort survival function





Source: HMD (2019). Calculations by the authors.