Examining Mortality Transition in Brass Logit Space

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Abstract

We examine mortality transition using Brass Logit Model. Building on Brass's (1968, 1971) observation that the age pattern of mortality in logit scale can be summarized by a simple linear relation with two parameters, one for level, and the other for shape, we define the Brass Logit Space as delineated by possible combinations of the two parameters. We apply the Brass Logit Model to the empirical life tables assembled by the Human Mortality Database to examine how mortality transition plays out in the Brass Logit Space. We find that mortality transition takes place in a restricted band within Brass Logit Space and proceeds with strong regularities. Our analysis shows that while mortality transition tends to be driven primarily by level changes, it is important to capture shape changes for accurate depiction for mortality pattern. We propose a modified Brass Logit Model for the purpose of diagnosis and projection.

Introduction: Brass Logit Model

Brass (1968, 1971) proposes a relationship model that captures the age pattern of mortality in logit scale with a simple linear model: for any two sets of observed survivorship functions in life tables l_x and l_x^s, $\text{logit}(1-l_x) = \alpha + \beta * \text{logit}(1-l_x^s)$, with radius $l_0=1$, and logit[1] is defined as $\ln((1-l_x)/l_x)$. As Brass (1971:70) states: "The simplicity of this system for describing relations among mortality patterns suggests that it depends on fundamental properties of the variation in death rates with age." The model provides a possibility to gain "new insights into the nature of the variation and providing a meaningful scale for its measurement."

The model is highly interpretable as the two parameters can be linked to different processes of mortality change: level and shape (slope). Moreover, such a parsimonious specification with simple arithmetic is attractive as the model can be easily adapted for many demographic applications, such as data examination, graduation, adjustment, estimation and projection. The model have shown to perform well with in many situations (Preston et al 2001, XXX).

One caveat of this model is that its accuracy depends on the choice of reference or standard mortality. Brass proposed two standards, labeled a general standard, and an African standard. "For an investigation of changes in mortality over time in a population the best base would be one of the life tables under study. Nevertheless for many purpose it is convenient to use a standard mortality schedule to which all others can be related." (Brass 1971:75-76) It is no surprise that deviations from linearity tend to be large when the observed mortality of a population is far from that of the standard. Zaba (1979) and Ewbank et al. (1983) extended the Brass logit model by adding additional parameters to better capture the mortality in childhood and adulthood. Murray et al. (2003) proposed a modified Logit system using l_5 and l_{60} and constructed a global standard based using 1802 "high-quality empirical life tables".

In this paper, we examine mortality transition using Brass Logit Model and Human Mortality Database. Building on Brass Logit Model, we first define the Brass Logit Space as delineated by possible combinations

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[1] In the original notation used by Brass (1971), logit is defined as $0.5*\ln((1-l_x)/l_x)^1$, . The multiplier .5 is dropped in this paper as this is nowadays how logit is defined in statistics texts and software packages.

of the two parameters. We then examine how mortality transition plays out in the Brass Logit Space using all the empirical life tables assembled by the Human Mortality Database. We find that mortality transition takes place in a restricted band within Brass Logit Space and proceeds with strong regularities. Our analysis shows that while mortality transition tends to be driven primarily by level changes, as revealed by Lee and Carter (1992) in their examination of the U.S. data, it is important to capture shape changes for accurate depiction for mortality pattern. We propose a modified Brass Logit Model for the purpose of diagnosis and projection.

Brass Mortality Space

Brass Logit Model defines a theoretical space of possible life table values: different combinations of α and β values can be used to produce variations of life tables, which we call as “Brass Mortality Space”. Figure 1 shows Brass Mortality Space illustrated in the contour of life expectancy at birth (e_0). The e_0 for Brass’s general standard life table is 43.1, as indicated by the e_0 value at alpha=0 and beta=1. It sets the limit of possible values of alpha and beta to produce empirically sensible life table values. As shown in Figure 1, both alpha and beta are confined in a relatively small range: between -6 and 2 for alpha, and between 0 and 2.5 for beta. The almost flat contour lines of e_0 when alpha is less than 0, shows that the effect of beta is relatively small unless it is very close to 0. At the same time, the effect of alpha depends largely on the value of beta: large when beta is close to 0, and small when beta is close to 2.5.

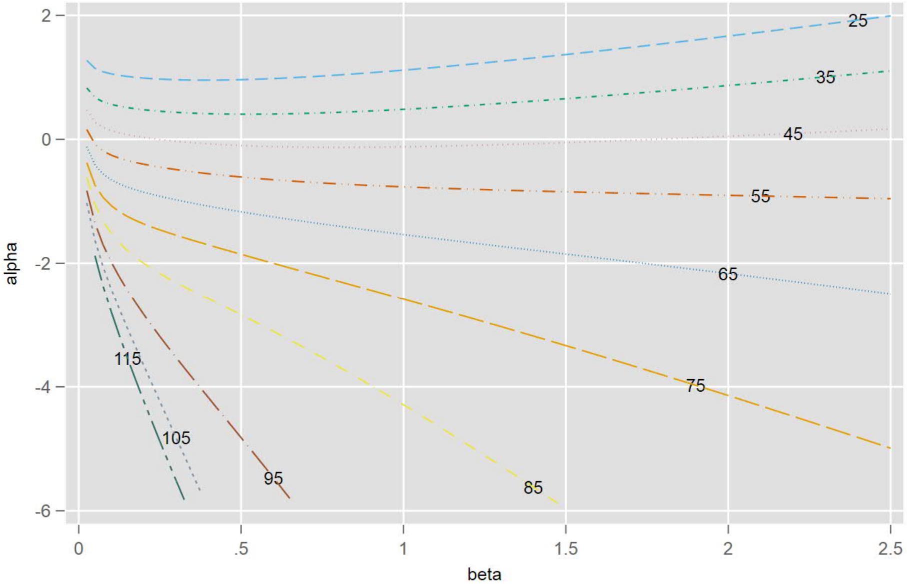

Figure 1. Brass Mortality Space: Combination of alpha, beta, and possible e0 values

How does the Brass Mortality Space match up with empirical data? Figure 2 projects Brass Logit Model parameters into Brass Mortality Space using 4,640 life tables compiled and published by the Human Mortality Database, with color/marker differentiating countries/regions. The empirical values are confined in a much smaller space. While there are some large variability in beta when alpha is less than 0 (and e0 less than 45), most of beta values are concentrated at a very small strip – 99% of all beta estimated are between 0.784 and 1.206, and 99% of alpha estimated are between 0.543 and -3.782. Moreover, there is a strong negative correlation between alpha and beta: Pearson correlation is -.68 for all the cases, and -.72 if limited to alpha < 0.

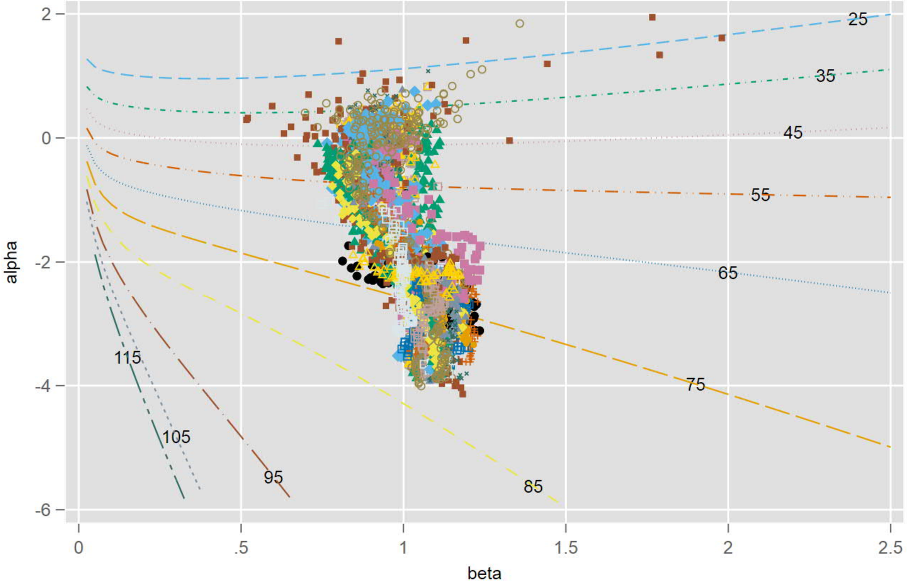

Figure 2. Human Mortality Database in Brass Mortality Space

To illustrate how mortality transition takes place in Brass Mortality Space, we use the longest series of life table. Figure 3 shows the history of Swedish mortality change from 1751 to 2017 in four panels. The top-left panel shows the change in the fitted alpha values. While alpha shows an overall decline trend, variability is much higher in early years than later years. The large variability in early years can be attribute to instability of mortality in early transition years. The alpha stays mostly above 0 until around 1850 when it started a gradual and almost monotonic decline, except for a visible spike around 1918. The top-right panel shows the fitted beta values over time. Just like alpha, its variability attenuates over time. Unlike the overall decline trend in alpha, the change in beta comes in stages. Before 1850, there is an overall trend of increase. Around 1850, the increase trend is reversed to an overall decline. The trend reversed again around 1900, and then again around 1950. The bottom left panel shows the combined effect of alpha and beta in the Brass Mortality Space, but we can now interpret it with time trend in our mind. The bottom right panel shows that Brass Logit Model fits Swedish data very well until up to about 1950 when R^2 started to drift below .99.

Figure 3 presents a highly interpretable process of mortality transition in Sweden. First, the goodness of fit statistics (R^2) shows the overall mortality age pattern remained relatively stable until around 1950, when a new mortality regime (that is different from Brass's standard model) takes shape. Second, mortality change in early years was erratic, as indicated both the large bounces in alpha and beta, and their combined effects in life expectancy at birth. Third, mortality change in Sweden is more dramatic in the changes of "level" than the changes in "shape" in Brass Mortality Space.

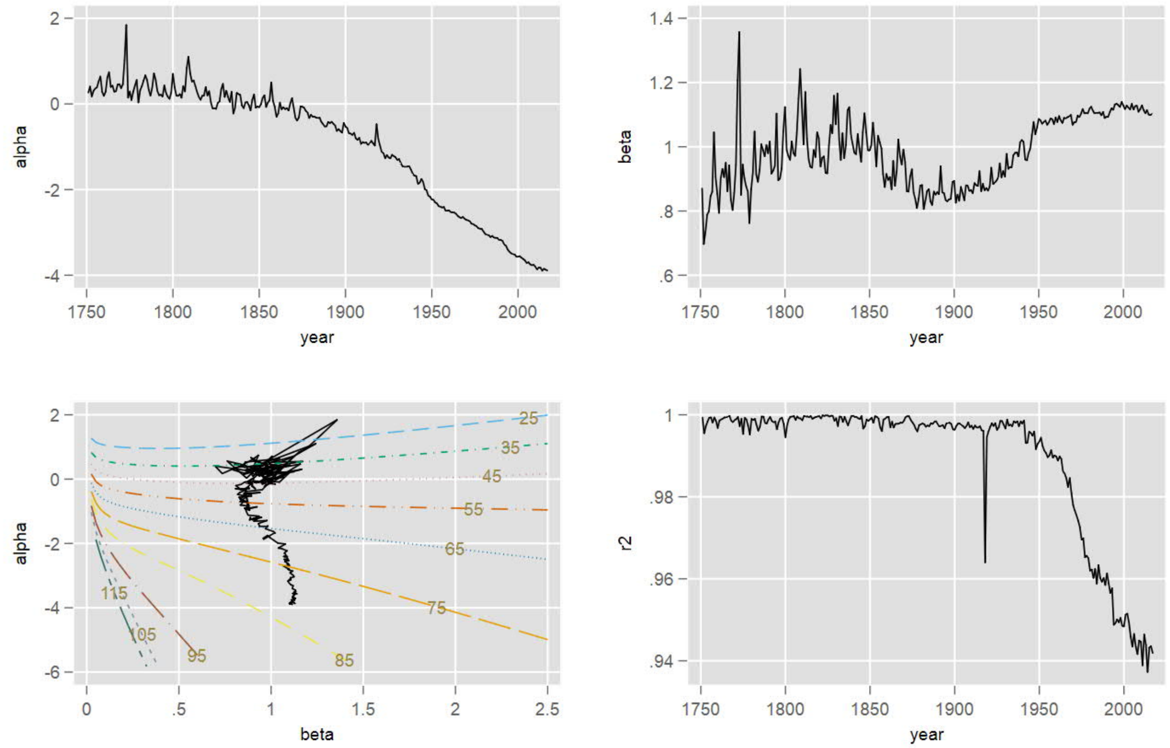

Figure 3. Swedish Mortality History in Brass Mortality Space

Their combined effects of alpha and beta suggests that there are 4 different stages of mortality transition in Sweden. At the first stage, from 1751 upto the first two decades of the 19th century, it is the both the stabilization of overall mortality level, and the decline of infant and child mortality as indicated by large variations in both alpha and beta, but an overall increase trend in beta. The second stage, from about 1825 to 1900, features gradual decline of overall mortality, and catchup of old age mortality as indicated by a decline alpha, and a decline beta. The third stage, from 1900 to 1950, features further decline of overall mortality, but once again the relative advantage of younger ages, with a major interruption of 1918 Spanish flue. The fourth stage, from 1950 to 2017, shows a continued gradual decline of mortality level, but the decline is largely balanced between younger ages and old ages. However, at this stage, because the Brass general standard life table has a much higher mortality, the model fits deteriorate, thus suggest a new mortality regime that is substantially different from Brass standard life table.

Because each country's mortality transition starts from different time, instead of index mortality transition by time, Figure 4 shows the process of mortality transition indexed on observed life expectancy at birth from all 49 regions in Human Mortality Database. The process of mortality change is highly consistent across 49 regions in terms of alpha, but comes with much larger variation in terms of beta. The variation of beta, while it is somewhat large at early stage of mortality transition (i.e. e_0<45), exists at all different levels of mortality level, thus suggesting that the regional difference in mortality variation shows no sign of convergence, even though all countries are moving towards high life expectancy. Just like in Sweden, the fits become not as good when life expectancy passing 60 – a level that many regions in the data has reached by 1950.

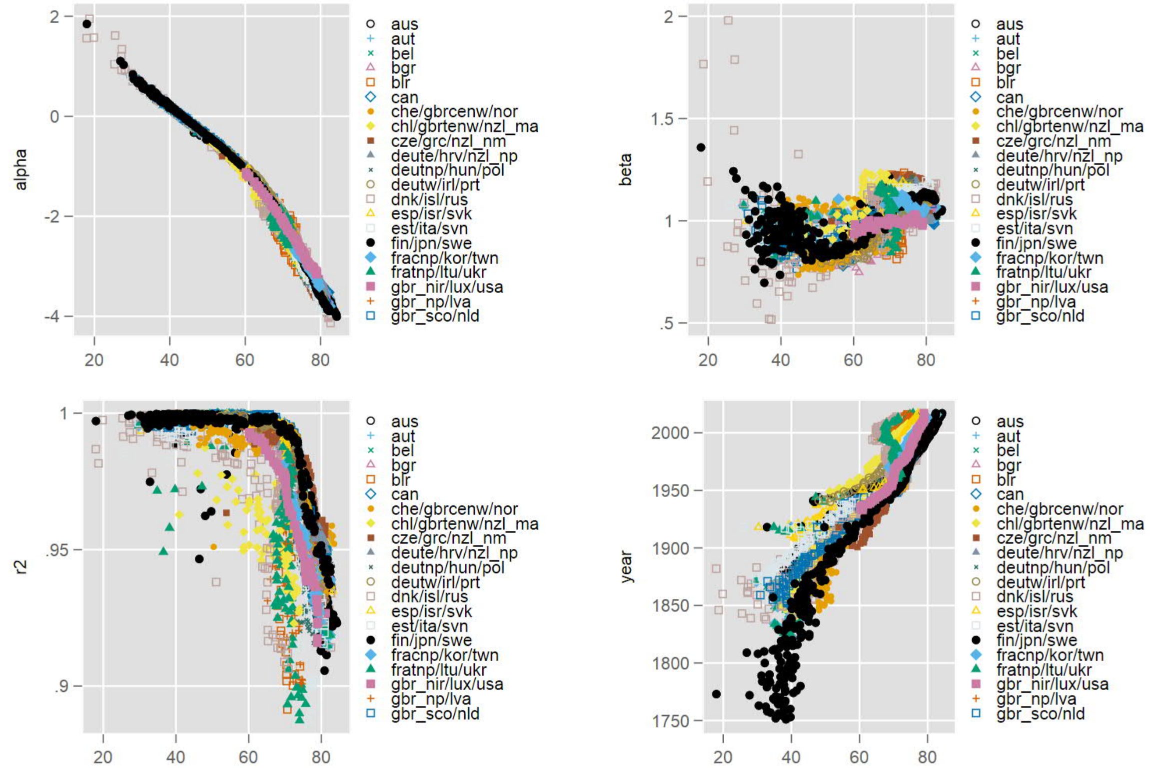

Figure 4. Mortality Transition in Brass Mortality Space: 49 regions from Human Mortality Database

The above analysis shows that the Brass Logit Model provides a very useful to analyze mortality transition. It reveals that mortality transition takes place in a restricted band within Brass Logit Space and proceeds with strong regularities. Our analysis shows that while large part of mortality transition can be captured by level change, it is important to consider shape change for more accurate depiction for mortality pattern. The large variation and lake of convergence in Brass beta confirm Brass's (1971) observation that it is best to region specific model for investigating of changes in mortality over time in a population. It also suggests a possibility of developing a modified Logit system that incorporates regional variation.

Modified Brass Logit Model

We propose a modified Brass Logit Model with one additional parameter that could be used capture the regional variation of mortality age pattern.