A Demographic Approach to the Analysis of Social Mobility*

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PRELIMINARY DRAFT

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Abstract

Most social mobility studies take a two-generation perspective, in which intergenerational relationships are represented by the association between parents' and offspring's socioeconomic status. This approach, albeit widely adopted in the literature, has serious limitations when more than two generations of families are considered. In particular, it ignores the role of families' demographic behaviors in moderating mobility outcomes and the joint impact of mobility and demography in shaping long-run family and population processes. This paper provides a demographic approach to the study of multigenerational social mobility, incorporating demographic mechanisms of births, deaths, and mating into statistical models of social mobility. Compared to previous mobility models for estimating the probability of offspring's mobility conditional on parent's social class, the proposed joint demography-mobility model treats the number of offspring in various social classes as the outcome of interest. This new approach shows the extent to which demographic processes may amplify or dampen the effects of family socioeconomic positions due to the direction and strength of the interaction between mobility and differentials in demographic behaviors. I illustrate a variety of demographic methods for studying multigenerational mobility with empirical examples using the IPUMS linked historical U.S. census representative samples (1850 to 1930) and the Panel Study of Income Dynamics (1968 to 2015), and simulation data that show other possible scenarios resulting from demography-mobility interactions.

1 INTRODUCTION

Studies on social mobility are dominated by a two-generation perspective, in which researchers analyze the extent to which one's socioeconomic status, in terms of education, income, occupations, and the like, is associated with that of one's parent (Blau and Duncan 1967; Breen 2004; Erikson and Goldthorpe 1992; Featherman and Hauser 1978; Hout 1983). The most common method of analysis uses mobility tables, a contingency-table technique that summarizes the *probability* a child will be in a certain social position given his parent's position (e.g., Ginsberg 1929; Glass 1954). From a statistical view, mobility tables are equivalent to a single transition matrix of a Markov chain, which describes the transition probability of moving from one social class to another in one generation step (Bartholomew 1967; Hodge 1966; Prais 1955; Svalastoga 1959; White 1963).

Mobility tables provide an elegant and effective approach to summarizing the transmission of social status across two generations, but this method's limitations are widely discussed and debated. Duncan (1966b: 17), for example, cautioned mobility researchers more than 50 years ago that

> *What is fundamental is that the process by which occupation structures are transformed—the succession of cohorts and intracohort net mobility—are not simply translatable into the processes one may observe in a so-called intergenerational occupation mobility table.*

Duncan did not explicitly use the phrase "demography," but his critique points to the importance of accounting for demographic processes that govern the transmission of social status from parents to offspring and the succession of generations in a population. More specifically, as Duncan (1966a) noted, the conventional mobility approach relies on a sample of respondents and their reports on their own parents. The parents are not representative of a previous generation or any cohort in "some definite prior moment in time" because the sample (1) necessarily omits individuals who never had children; (2) overrepresents parents who have many offspring; and (3) includes parents born into different birth cohorts who vary by childbearing age.

From a demographic perspective, generations within families are linked not only by their socioeconomic statuses, but also by their fertility, mortality, and marriage, among other demographic behaviors. These demographic outcomes, often stratified by social class, lead to variations among families in resources allocation, household formation, and changes in kinship structure, which, in turn, limit and condition the amount of family capital that can be inherited by subsequent generations (see examples in Lam 1986; Mare 1997; Mare and Maralani 2006; Maralani 2013; Preston and Campbell 1993). Compared to the traditional approach based on mobility tables, the demographic approach provides a more complete account of intergenerational processes, shifting attention from "how likely that offspring's status resembles that of their parents" to "how intergenerational effects come about" (Mare 2015: 101). By doing so, researchers are no longer restricted to analysis of parents and offspring *conditional on the existence of a given offspring*, but are now also able to consider the degree to which offspring will come into existence as an integral part of intergenerational influences (Mare and Maralani 2006). The demographic view of social mobility, albeit long established in the literature (e.g., Matras 1961, 1967), has been largely overlooked by major studies on social mobility until recently (Breen and Ermisch 2017; Lawrence and Breen 2016; Maralani 2013; Mare 1997; Mare and Maralani 2006).

The present study generalizes the demographic approach, which has hitherto focused on social mobility between two generations, to multiple generations. Multigenerational mobility research has proliferated in recent years, with new studies leveraging the increased availability of longitudinal, genealogical, and linked administrative data that provide information on family members over three or more generations (reviewed in Ruggles 2014; Ruggles et al. 2015; Song and Campbell 2017). Yet, most of these studies follow the tradition of mobility tables, examining the association of social status across three generations, especially the role of grandparents on grandchildren in status attainment, net of the widely-studied effects of parents (Chan and Boliver 2013; Ferrie et al. 2016; Jæger 2012; Mare 2011, 2014; Pfeffer 2014; Song 2016; Zeng and Xie 2014). Despite the considerable merit of these studies, the complexity of multigenerational influences has not been fully explored. To pass on their advantages or disadvantages, families must first have at least one offspring in each generation who can carry the family legacy. In the long run, families' demographic behaviors may mute or exacerbate the effect of social mobility, leading to varying numbers and types of offspring among families. Eventually, some families may grow and account for a disproportionately large share of the population after several generations or hundreds of years, whereas others may decline or even become extinct (Song, Campbell, and Lee 2015). This paper illustrates multigenerational models that not only account for the circulation of elites in a society due to social immobility, but also provide aggregate-level inferences about long-term population renewal and change.

Building on previous theoretical constructs and contributions from social mobility and population renewal models, I introduce several joint demography-mobility models. The models incorporate a few new features into conventional discrete-state Markov chain mobility models (Bartholomew 1967; Blumen et al. 1955; Hodge 1966; Matras 1961; Singer and Spilerman 1973) by (1) incorporating multigenerational effects; (2) combining demographic processes with the transmission of social status; (3) addressing population heterogeneity in social mobility; and (4) differentiating between one-sex and two-sex approaches. These models are extensions to the two-generation social reproduction model that focuses on female populations in Mare and Maralani (2006).

The rest of the paper proceeds as follows: In section 2, I describe traditional methods based on discrete-time Markov chains, in which time is measured as "generations" and social status is measured by a finite number of discrete, qualitatively different categories.[1] Section 3 introduces a joint demography-mobility model, also known as the social reproduction model, which reflects the evolution of socioeconomic distributions over generations in a population. It also provides examples of higher-order social reproduction models that include additional parameters for ancestral influences. Section 4 introduces various definitions of multigenerational effects based on models in Section 3 and shows how to decompose the effects into demographic and mobility components. Section 5 shows long-term equilibria of multigenerational social reproduction models compared to those implied by simple Markov models. Section 6 illustrates a mixture social reproduction model that allows for heterogeneous mobility regimes among subpopulations. Section 7 describes a two-sex version of the multigenerational social reproduction model that accounts for interactions between males and females, namely, the process through which two sexes mate and produce offspring with others of similar social statuses and jointly influence the social mobility outcomes of their offspring. Section 8 provides empirical examples of various types of multigenerational mobility and demographic models using data from the IPUMS linked representative samples of U.S. census data

[1] I will not discuss continuous-time Markov chain models, which require extensive information about mobility measured in "real" time (Blumen et al. 1955; Goodman 1961; Singer and Spilerman 1976, 1977; Spilerman 1972b). Also, this paper does not address models that rely on continuous measures of social status. These models, exemplified by the path analysis used in Blau and Duncan (1967), often focus on answering questions related to the determinants of social status rather than the overall extent of social mobility.

(IPUMS linked, 1850 to 1930), the Panel Study of Income Dynamics (PSID, 1968 to 2015), and simulation data that show a range of hypothetical demography-mobility interactions. Section 9 concludes the paper by identifying areas for future research on multigenerational methodology.

2 CLASSICAL SOCIAL MOBILITY MODELS BASED ON MARKOV CHAINS

From the outset of studies on social mobility, important theoretical and empirical advances have accompanied the development of new methods of data collection, measurement, and analysis (Ganzeboom et al. 1991). In one of the earliest studies on social mobility, Prais (1955) shows that the representation of mobility processes as Markov chains has methodological advantages over contingency tables (e.g., Ginsberg 1929; Glass 1954). The Markov chain is a simple form of stochastic modeling, in which the outcome state of the present generation depends only on that of the parent generation, not any other preceding generation. The model provides new measures of mobility—such as equilibrium distribution of social classes and the average time spent in a social class—beyond measures used in contingency tables, such as vertical and horizontal mobility rates (Sorokin 1959 [1927]), inflow and outflow percentages (Lipset and Bendix 1959), and mobility ratios (Carlsson 1958; Glass 1954; Rogoff 1953; Tyree 1973). These early endeavors, widely considered to be the first generation of mobility research, all rely on descriptive, global measures to summarize mobility patterns (Ganzeboom et al. 1991; Boudon 1973).

Below, I provide a brief overview of classic mobility models based on Markov chains. These models typically start with a mobility table, in which rows refer to fathers' positions and columns refer to sons' positions (with I and J categories, respectively, and typically, $I = J$) (Bartholomew 1967). Mobility tables can be converted into a Markov chain transition matrix by standardizing mobility rates between categories as follows:

$$\sum_{j=1}^{J} p_{Y_2=j|Y_1=i} = \sum_{j=1}^{J} \frac{n_{ij}}{n_{i+}} = 1 \tag{1}$$

where $p_{Y_2=j|Y_1=i}$ denotes the probability that the son (G2) of a father (G1) in social position i ends up in position j; n_{ij} denotes the number of father-son dyads in positions i and j; and n_{i+} denotes the total number of fathers in position i regardless of their sons' positions.

Suppose we observe f_i fathers in social position i and s_j sons in position j. The transition matrix $\mathbf{P}$ that transforms the distribution of fathers into the distribution of sons satisfies

$$s_j = \sum_{i=1}^{I} f_i \cdot p_{Y_2=j|Y_1=i} \quad (j = 1, 2, ..., J). \tag{2}$$

In matrix notation, fathers and sons in different positions are denoted by vectors $\mathbf{F} = [f_1, f_2, ..., f_i, ..., f_n]$ and $\mathbf{S} = [s_1, s_2, ..., s_i, ..., s_n]$, respectively. The matrix of mobility probabilities $\mathbf{P}$ with $p_{j|i}$ in the ith row and jth column is represented as a square matrix in the following form:

$$\mathbf{P} = \begin{bmatrix} p_{1|1} & p_{2|1} & \cdots & p_{n|1} \\ & & \cdots & \\ \vdots & & p_{j|i} & \vdots \\ & & \cdots & \\ p_{1|n} & p_{2|n} & \cdots & p_{n|n} \end{bmatrix}_{n \times n} \tag{3}$$

A transition matrix has all entries as mobility probabilities between 0 and 1. The sum of entries in each row equals 1. The matrix shows the probability of change in social position from one generation to the next. Table 1 displays a transition matrix based on occupation groups from the IPUMS linked representative samples using historical censuses 1850 to 1930. For example, if a father is in the farming occupation, then the probability that his son will be in the same occupation is 0.764, and the probability that his son will be in an upper nonmanual occupation is 0.064. We can further divide individuals in each cell of the mobility matrix into subgroups that vary by their characteristics relevant for mobility and estimate the mobility probability in each cell using regression techniques (Sørensen 1975; Spilerman 1972a).

The matrix form of the intergenerational transmission of social classes is written as

$$\mathbf{S}_{1 \times n} = \mathbf{F}_{1 \times n} \mathbf{P}_{n \times n}. \tag{4}$$

Assuming mobility rates are fixed over time, we can derive the distribution of men after two generations as

$$\mathbf{S}^{(2)} = \left(\mathbf{F}^{(0)} \cdot \mathbf{P}\right) \cdot \mathbf{P} = \mathbf{F}^{(0)} \cdot \mathbf{P}^2. \tag{5}$$

Furthermore, the distribution of descendants, namely the expected proportion of men in various social positions after t generations, can be projected by taking the matrix to the t^{th} power:

$$\mathbf{S}^{(t)} = \mathbf{S}^{(t-1)} \cdot \mathbf{P} = \mathbf{F}^{(0)} \cdot \mathbf{P}^{(t)}. \tag{6}$$

This equation shows the process through which the initial progenitor distribution is transformed into subsequent generations after a number of generations of social mobility. The process retains no memory, in the sense that a man's social position entirely depends on that of his father. If the position of one's father has been taken into account, then his grandfather, great-grandfather, and earlier ancestors have no impact on his probability of attaining a specific position. Once a grandfather fails to transmit his position to his son, he is incapable of influencing the outcomes of his grandson independently of his son. The memoryless property also makes it possible to predict how the Markov process behaves in the long run—that is, the eventual distribution of descendants after a sufficient number of generations. Provided that the transition matrix is *regular*, as time progresses, the process will "forget" its initial distribution and converge to a unique equilibrium distribution of the descendants that is unrelated to the initial distribution (Norris 1998).[2] This property implies that

$$\lim_{t \to \infty} \mathbf{F}^{(0)} \cdot \mathbf{P}^{(t)} = \pi \tag{7}$$

where π is called the equilibrium vector of the Markov chain. This property suggests that in the short run, the initial distribution of progenitors influences future generations, but the influence diminishes as time passes. In the long run, the descendant distribution is only determined by the transition matrix $\mathbf{P}$. Section 5 further discusses how to obtain the vector π by solving the Markov chain equilibrium.

According to Coleman (1964a: 462), "the intent of the (Markov) model is not to mirror reality in all its facets. It is, instead, to see just *how much* of reality can be mirrored by a highly constrained process. That is, our question will be: How well does this rather restrictive assumption allow us to account for the data on intergenerational mobility?" To evaluate the suitability of a Markov chain

[2] Regular means all entries in some power of the transition matrix are positive. Or more strictly speaking, a Markov chain is irreducible, positive recurrent, and aperiodic.

model for representing a multigenerational process, it is important to first identify assumptions implied in this model. Below, I list five key assumptions that are modified from Pullum (1975: 16–17).

Assumption 1 [No Demography]. Families' social mobility, $\mathbf{P}_{Y_2|Y_1}$, is assumed to be independent of demographic behaviors in any generation, $\mathbf{R}$, such as mortality, fertility, adoption, mating, and migration, as well as the timing of these events. In particular, families' social status, $\mathbf{X}$, does not affect their number of children or long-term reproductive success.

Assumption 2 [Markov Property]. All multigenerational influences on a son are mediated by the father, $\mathbf{P}_{Y_n|\bar{\mathbf{Y}}_{n-1}} = \mathbf{P}_{Y_n|Y_{n-1},Y_{n-2}\cdots Y_1} = \mathbf{P}_{Y_n|Y_{n-1}}$. The grandfather, great-grandfather, remote ancestors, and wider kin network have no effect on the son when the father's influence is accounted for. Thus, the total influences of one's ancestors are equal to the total influence of the father.

Assumption 3 [Transition Stationality]. The intergenerational transition matrix does not change as the history unfolds, that is, $\mathbf{P}(t) = \mathbf{P}$. All multigenerational relationships can be derived from the time-invariant two-generation mobility table.

Assumption 4 [Homogeneous Mobility Regime]. A single mobility regime, $\mathbf{P}$, in the society is assumed, so that all individuals in a population are subject to the same set of mobility probabilities given their fathers' social class, $p_{Y_2|Y_1}$. This assumption also implies that the population is homogeneous with respect to characteristics other than the measure of social class under consideration.

Assumption 5 [One-Sex Mobility]. The model includes only fathers and sons; it ignores women's social statuses and the potential influence of mothers and maternal ancestors, namely, $\mathbf{P}_{Y_{gn=\text{son}}|Y_{g_{n-1}=\text{father}}} = \mathbf{P}_{Y_{gn=\{\text{son, daughter}\}}|Y_{g_{n-1}=\{\text{father, mother}\}}}$. The role of mating rules, such as assortative mating by social status in determining the number of marriages and families' reproductive behaviors in a population, is not considered.

Mare (2011) provides examples of social contexts in which these assumptions may be violated and discusses the implications of these violations for understanding multigenerational mechanisms. In the following sections, I modify each of the five assumptions and show variants of stochastic models that may better characterize multigenerational processes under different circumstances.

3 A JOINT DEMOGRAPHY-SOCIAL MOBILITY MODEL

3.1 A Two-Generation Setup

The mobility table in Markov chain models provides a straightforward way of assessing the degree of social mobility between generations. Yet, as discussed earlier, mobility tables represent fathers' and sons' occupational distributions by giving equal weight to sons from families of unequal size, ignoring the fact that some fathers may have many sons while others have none. The transmission of social status from fathers to sons is not a simple, one-to-one mapping; instead, demographic processes, such as births, deaths, and migrations, may all influence the number of offspring observed in a mobility table (Kahl 1957; Pullum 1970).

So far, we do not have an agreed-upon solution for translating a Markov mobility model into a model that illustrates changes in social structure and population renewal simultaneously. Conceptually, the Markov model is flawed by the lack of a distinction between generation and birth cohort. If we define the son generation as a birth cohort whose occupational distribution is observed at a recent point in time, then fathers' occupations—represented by the marginal distribution of the mobility table—do not comprise the occupational distribution of any birth cohort at any prior point in time (Duncan 1966a). Fathers' levels and timing of fertility vary, so a generation of fathers consists of a group of men whose birth years are not well-defined—and often not even asked in retrospective surveys of sons. Such ambiguity also occurs in the definition of the sons' birth cohort when mobility tables are constructed from a prospective perspective (Song and Mare 2015; Yasuda 1964). Methodologically, a mobility table is not equivalent to a population projection matrix that can be used to describe population dynamics. Mobility tables often include no age-specific information that could be used to predict the progression of birth cohorts. Nor do they include individuals' life history events—such as the school-to-work transition, job promotion and changes, retirement, or even death—that could be used to predict occupational compositions of fathers and sons in the labor market. Despite all these potential difficulties in combining a mobility model with demographic components, a few studies propose variants of Markov models that allow for differential demographic rates (Chu and Koo 1990; Matras 1961, 1967; Mare 1997; Mare and Maralani 2006; Maralani 2013; Preston 1974; Preston and Campbell 1993; Lam 1986, 1997). These models provide a good starting point for future work.

To relax **Assumption 1**, Matras (1961) first proposed a Markov model that incorporates differential population growth as follows:

$$s_j = \sum_{i=1}^{I} f_i \cdot r_i \cdot p_{Y_2=j|Y_1=i} \quad (j = 1, 2, ..., J). \tag{8}$$

where f_i denotes the number of fathers in social class i; s_j denotes the number of sons in class j; r_i denotes the expected number of sons born to a man in class i who survive to adulthood or are old enough to acquire a social position;[3] and $p_{Y_2=j|Y_1=i}$ denotes the probability that a son born to a father in class i will attain class j.[4]

Relying on the recursive form of the model, we can model the socioeconomic distribution of descendants given that intergenerational fertility and mobility processes are fixed over time (namely, a time-homogeneous Markov chain). Set a diagonal matrix for the differential fertility component, $\mathbf{R} = [r_{ij}]$, where $r_{ij} = r_i$ for $i = j$ and $r_{ij} = 0$ for $i \neq j$. $\mathbf{P}$ is the same mobility matrix defined in equation (3). Let $\mathbf{R} \cdot \mathbf{P} = \mathbf{C}$, and we obtain the following intergenerational relationship:

$$\mathbf{S}^{(2)} = \mathbf{S}^{(1)} \cdot \mathbf{C} = \mathbf{F}^{(0)} \cdot \mathbf{C}^2. \tag{9}$$

In general, the generation-to-generation change is represented by

$$\mathbf{S}^{(t)} = \mathbf{F}^{(0)} \cdot \mathbf{C}^t. \tag{10}$$

Subsequent work has extended this basic model in several ways. Matras (1967) introduced a model that incorporates the age structure of each generation, which was later analyzed empirically by Lam (1986) and Mare (1997). Preston (1974) developed a model that separates white and non-white families. Mare and his collaborators further decomposed the differential reproduction rates into marriage, fertility, and mortality components (Kye and Mare 2012; Maralani 2013; Mare and Maralani 2006; Maralani and Mare 2005; Mare and Song 2014).

[3]In population data, the average number of sons who survived to adulthood may not be available. An approximate measure is the Gross Reproduction Rate, namely, the average number of sons who would be born to a man during his lifetime if he lives through his child-bearing years and conforms to the age-specific reproduction rates of a given year.

[4]Matras (1961) uses the proportion of fathers (sons) in each occupation, but here we use the number of fathers (sons) to be consistent with equation (8).

Overall, these models show the effect of a person's social class in one generation on the expected number of sons in various social classes in the next generation—that is, they show the joint effects of a man's social class on his demographic behaviors and his children's socioeconomic attainment. Therefore, these models illustrate the transformation from **F** to **S** as a sociodemographic process rather than strictly a social mobility process. In subsequent sections, I refer to these models as social reproduction models or sociodemographic mobility models.

As discussed earlier, the model specifications in equation (8) may simplify demographic processes in social mobility, especially by relying on the concept of generation rather than using a real-time scale (Duncan 1966a). Projections from the model often do not mirror observed empirical processes that evolve continuously in time. Yet, taken qualitatively, conclusions from these models may still reflect general trends in family dynamics in the long run.

3.2 Multigenerational Models

One central assumption of the Markov chain model is that each generation directly influences only the immediately following generation, exerting no direct effect beyond its offspring (**Assumption 2**). No matter how much influence parents have on their children's outcomes, they do not influence their grandchildren's outcomes independently of their own children. This means the social system has no memory: if a family loses its existing advantages, it has to start from scratch. This assumption may be invalid under some social circumstances, such as when multigenerational social processes are non-Markovian. In particular, individuals' social mobility may depend on various forms of multigenerational influence, such as direct influences from grandparents and great-grandparents, cumulative advantages (or disadvantages) of prior generations, legacy influences of remote ancestors who experienced extreme hardship or success, or supplementary influences of nonresident kin in extended families (Mare 2011). Some of these processes can be represented by second- or higher-order Markov chains (see, e.g., Goodman (1962) on attitude change and Hodge (1966) on three-generation mobility), but in general, I will refer to them as non-Markovian generational processes.

The simplest way to relax the Markovian assumption is to incorporate the effect of grandparents into Matras's model shown in equation (8):

$$s_j = \sum_i \sum_k f_{ik} \cdot r_{ik} \cdot p_{Y_3=j|Y_2=i,Y_1=k} \tag{11}$$

where s_j denotes the number of men in the offspring generation who are in class j ($j = 1, ..., J$); f_{ik} denotes the number of men in the paternal generation who were in class i and whose fathers were in class k; r_{ik} denotes the expected number of sons born to each man in f_{ik}; and $p_{Y_3=j|Y_1=i,Y_2=k}$ denotes the probability that a son with a father in class i and a grandfather in class k will attain class j.

We can extend Matras's model by adding more demographic parameters and families' socioeconomic characteristics from prior generations:

$$s_{j|ikl,c} = f_{ikl,c} \cdot m_{ikl,c} \cdot r_{ikl,c} \cdot p_{Y_n=j|Y_{n-1}=i,Y_{n-2}=k,Y_{n-3}=l,\bar{Y}=c} \tag{12}$$

where $s_{j|ikl,c}$ denotes the number of men in the offspring generation who are in class j ($j = 1, ...J$) and whose fathers were in class i ($i = 1, ..., I$), grandfathers were in class k ($k = 1, ..., K$), and great-grandfathers were in class l ($l = 1, ..., L$); $m_{ikl,c}$ denotes the probability of getting married (or the average number of marriages) for men in $f_{ikl,c}$ in the parent generation; and $r_{ikl,c}$ denotes the expected number of sons born to each marriage of men in the parent generation. The extra subscript c ($c = 1, ..., C$) refers to this person's ancestral traits that do not change over generations (e.g., an indicator of remote family history of slavery or royalty). More generally, if the model parameters depend on the socioeconomic status of all prior generations and let $\bar{\mathbf{Y}}_{n-1} = \{Y_1, Y_2, \cdots, Y_{n-1}\}$, the model can be written as

$$s_{Y_n} = \sum_{Y_1} \cdots \sum_{Y_{n-1}} f_{\bar{\mathbf{Y}}_{n-1}} \cdot m_{\bar{\mathbf{Y}}_{n-1}} \cdot r_{\bar{\mathbf{Y}}_{n-1}} \cdot p_{Y_n|\bar{\mathbf{Y}}_{n-1}} \tag{13}$$

To predict the number of descendants in the n^{th} generation, we rely on the recursive relationship shown in equation (10); the resulting model is written as

$$s_{Y_n}^{(n)} = \sum_{Y_1} \cdots \sum_{Y_{n-1}} f_{Y_1} \cdot m_{Y_1} \cdot r_{Y_1} \cdot p_{Y_2|Y_1} \cdot m_{\bar{\mathbf{Y}}_\mathbf{2}} \cdot r_{\bar{\mathbf{Y}}_\mathbf{2}} \cdot p_{Y_3|\bar{\mathbf{Y}}_\mathbf{2}} \cdots m_{\bar{\mathbf{Y}}_{n-1}} \cdot r_{\bar{\mathbf{Y}}_{n-1}} \cdot p_{Y_n|\bar{\mathbf{Y}}_{n-1}}$$

$$= \sum_{Y_1} \cdots \sum_{Y_{n-1}} f_{Y_1} \cdot \prod_{i=1}^{n-1} m_{\bar{\mathbf{Y}}_i} \cdot r_{\bar{\mathbf{Y}}_i} \cdot p_{Y_{i+1}|\bar{\mathbf{Y}}_i} \tag{14}$$

The marriage (m), fertility (r), and social mobility (p) terms can be modeled by generalized linear models as functions of independent variables. Marriage outcomes are often assumed to be dichotomous if the probability of getting married is considered, or non-negative counts if the number of marriages is considered. The latter applies to populations that have high rates of multi-partner fertility or polygamy. The marriage term thus can be characterized by a logit or negative binomial function. Reproduction outcomes are often assumed to follow a Poisson distribution with a possible overdispersion parameter and modeled by the negative binomial function. The mobility term can be modeled by multinomial logistic models when we assume multiple categories of social statuses. This model is restricted to influences of the father, grandfather, and great-grandfather, but similar recursive models can incorporate influences from more generations or from paternal and maternal sides of the family. In general, assume the response variable $\mathbf{X}$ (either m, r, or p) is generated from a particular distribution in the exponential family, such as binomial, Poisson, or multinomial distributions, among others. The mean, $\boldsymbol{\mu}$, of the distribution depends on the independent variables, $\mathbf{Z}$, which may include socioeconomic status measures of more than one generation within a family:

$$\mathbb{E}(\mathbf{X}) = \boldsymbol{\mu} = g^{-1}(\mathbf{Z}\boldsymbol{\beta}), \tag{15}$$

where $\mathbb{E}(\mathbf{X})$ is the expected value of $\mathbf{x}$; $\mathbf{z}\boldsymbol{\beta}$ is the linear predictor, a linear combination of unknown parameters $\boldsymbol{\beta}$; and g is the link function.

Despite the importance of validating the Markovian assumption in mobility models, only a few studies have tested the assumption empirically (Hodge 1966; Warren and Hauser 1997; see a review of these studies in Appendix Table S1). The increasing availability of longitudinal data in recent years has facilitated a growing body of scholarship that investigates the Markovian assumption more thoroughly using empirical evidence from the United Kingdom (Chan and Boliver 2013), the United States (Jæger 2012; Song 2016; Wightman and Danziger 2014), Germany (Hertel and

Groh-Samberg 2014), the Netherlands (Bol and Kalmijn 2016; Knigge 2016), Sweden (Hällsten 2014), Denmark (Møllegaard and Jæger 2015), Finland (Erola and Moisio 2007), mainland China (Zeng and Xie 2014), and Taiwan (Chiang and Park 2015). Findings from these studies are far from conclusive, suggesting the validity of the Markovian assumption may vary across time and social context. Even within a society, patterns of social mobility may vary by the form of social status under consideration, be it stocks of social advantages, such as business, land, or estate ownership, or flows of advantages such as income, occupation, and education. Moreover, any test of the Markovian assumption may be subject to the "lumpability" problem (Kemeny and Snell 1960: pp.123-139): a non-Markovian chain may become Markovian if we combine or divide some of the transition states. Therefore, any conclusion regarding the mobility pattern is valid only under the condition that the states are defined the way they are (McFarland 1970).

3.3 Age-Classified Models

Regular mobility models often ignore the age structure of the parent or the offspring generation. Such a simplification does not affect our understanding of the long-term behaviors of a Markov chain—namely, the chances individuals will achieve a certain social class conditional on their parent's or ancestor's social status. Yet the distribution of fathers or sons, even after the reproduction factor described in the previous section is accounted for, only reflects the overall size of each generation, not the population structure at a given point in time. From a demographic perspective, all accurate representations of population growth—or "transformations of occupation structure" (Duncan 1966a)—depend on age-specific fertility and mortality rates. In his classic work on population projection, P. H. Leslie (1945: 183) showed that "the age distribution of the survivors and descendants of the original population at successive intervals of time" can be derived from simple matrix multiplication, assuming the regime of mortality and fertility is time-constant or year-to-year change in mortality and fertility is known. Keyfitz (1964) introduced this method to the study of human populations. Specifically, let $r_{i,t}$ refer to age-specific fertility rates, often based on five-year age groups, for social class i and age group t; $r_{i,t}$ is a positive number for men within the reproductive age range and zero otherwise. In addition, let $\frac{{}_5L_{i,t+5}}{{}_5L_{i,t}}$ refer to the life table function of surviving from age t to $t+5$ for social class i. The social reproduction models shown in equation

(8) thus can be represented as

$$s_{j,1} = \sum_{i=1}^{I} \sum_{t=1}^{T} f_{i,t} \cdot r_{i,t} \cdot p_{Y_2=j|Y_1=i} \quad (j = 1, 2, ..., J) \tag{16}$$

$$s_{j,t+5} = s_{j,t} \cdot \frac{{}_5L_{j,t+5}}{{}_5L_{j,t}} \tag{17}$$

$$f_{i,t+5} = f_{i,t} \cdot \frac{{}_5L_{i,t+5}}{{}_5L_{i,t}} \tag{18}$$

Note that this model assumes social attainment is completed at births, and no intragenerational mobility is allowed for either the father or son generation. Predictions based on these assumptions may detract from the exact number of incumbents in each social class, but this will not affect conclusions regarding the overall social trend from an intergenerational perspective. The matrix forms of similar models based on the Leslie matrix are described in Matras (1967) and Mare (1997).

4 SOCIAL MOBILITY EFFECT VS. DEMOGRAPHIC EFFECT

Using the models described above, we can estimate the effect of one generation on the next or several generations later in terms of (1) the pure mobility effect based on the classic Markov models, and (2) the joint mobility and demography effect based on the social reproduction model in equation (8). The effects are defined in ratio measures and difference measures. The ratio measure refers to arithmetic quotients of mobility or demographic outcomes between two types of families; the difference measure refers to the difference score between the two. Both measures are widely used in social sciences (see a recent review and critique by Stolzenberg 2018), but ratio measures are more popular in the literature of social mobility, especially in mobility models based on log-linear analysis and odds ratios (Agresti 2013; Powers and Xie 2000). This section illustrates how to quantify various components of mobility and demography effects using the following definitions and decomposition methods.

4.1 Net and Total Mobility Effects

In traditional mobility models, researchers typically measure mobility by estimating differences in the probability of achieving high social status for children who grew up in rich versus poor families. For example, Chetty et al. (2014) show that for individuals born in 1971, the probability of reaching the top fifth income quintile group conditional on a parent (either father or mother) being in the top quintile is 31.1%, compared to 8.4% for individuals whose parents' income is in the bottom quintile. Based on a ratio measure, the total mobility effect (TME) of having a parent in social class k relative to social class j for children to attain social class k can be defined as

$$\text{TME}^{P} = p_{Y_2=k|Y_1=k} \big/ p_{Y_2=k|Y_1=j} \tag{19}$$

where $p_{Y_2=k|Y_1=k}$ (or $p_{Y_2=k|Y_1=j}$) refers to the probability that children whose parents are in class k (or j) will end up in class k.

Likewise, the total mobility probability effect of having grandparents in social class k relative to class j is estimated by combining the mobility from grandparents to parents and from parents to offspring:

$$\text{TME}^{GP} = \sum_i p_{Y_3=k|Y_2=i,Y_1=k} \cdot p_{Y_2=i|Y_1=k} \Big/ \sum_i p_{Y_3=k|Y_2=i,Y_1=j} \cdot p_{Y_2=i|Y_1=j} \tag{20}$$

where $p_{Y_3=k|Y_2=i,Y_1=k}$ (or $p_{Y_3=k|Y_2=i,Y_1=j}$) refers to the probability that children whose parents are in class i and grandparents are in class k (or j) will end up in class k; and $p_{Y_2=i|Y_1=k}$ (or $p_{Y_2=i|Y_1=j}$) refers to the probability that children whose parents are in class k (or j) will attain class i.[5] If we control for the parent generation, the mobility ratio is defined as the net mobility effect (NME):

$$\text{NME}^{GP} = p_{Y_3=k|Y_2=i,Y_1=k} \Big/ p_{Y_3=k|Y_2=i,Y_1=j}. \tag{21}$$

The total and net effects of grandparents are often unequal, but they are the same for the effect of the parent generation defined in equation (19).

4.2 Net and Total Social Reproduction Effects

Based on the social reproduction model discussed above, we define the total and net Social Reproduction Effects (SRE) for a targeted social class category relative to a baseline category in each generation that affects an individual's own social class. For example, the expected number of individuals in class category k whose parents were in class k relative to those whose parents were in

[5]Intergenerational mobility is assumed to be time-invariant, so $p_{Y_2=i|Y_1=k}$ refers to the same probability from the grandparent to the parent generation in equation (20) and from the parent to the offspring generation in equation (19).

class j is[6]

$$\text{SRE}_{k|j} = \frac{s_{Y_2=k|Y_1=k}}{s_{Y_2=k|Y_1=j}} = \frac{f_k \cdot r_k \cdot p_{Y_2=k|Y_1=k}}{f_j \cdot r_j \cdot p_{Y_2=k|Y_1=j}}. \tag{22}$$

If we consider multiple generations, we can further differentiate between the net and the total effect of a prior generation on the present generation. The net social reproduction effect (NSRE) of grandparents is defined as the comparative advantage of a parent in class j and a grandparent in class k over a parent and a grandparent both in class j in producing grandchildren in class k:

$$\text{NSRE}^{GP}_{k|j} = \frac{s_{Y_3=k|Y_2=j,Y_1=k}}{s_{Y_3=k|Y_2=j,Y_1=j}} = \frac{f_{jk} \cdot r_{jk} \cdot p_{Y_3=k|Y_2=j,Y_1=k}}{f_{jj} \cdot r_{jj} \cdot p_{Y_3=k|Y_2=j,Y_1=j}}. \tag{23}$$

Alternatively, we can define the NSRE of grandparents by fixing parents' social class at k. The two NSRE definitions will lead to the same estimates if there are no interaction effects between grandparents' class and parents' class in determining levels of r and p.[7]

By contrast, the total social reproduction effect is the comparative advantage of a grandparent in position k over a grandparent in position j in producing grandchildren in position k regardless of the parent's position. Specifically,

$$\text{TSRE}^{GP}_{k|j} = \frac{s^{(2)}_{Y_3=k|Y_1=k}}{s^{(2)}_{Y_3=k|Y_1=j}} = \frac{\sum_i f_k \cdot r_k \cdot p_{Y_2=i|Y_1=k} \cdot r_{ik} \cdot p_{Y_3=k|Y_2=i,Y_1=k}}{\sum_i f_j \cdot r_j \cdot p_{Y_2=i|Y_1=j} \cdot r_{ij} \cdot p_{Y_3=k|Y_2=i,Y_1=j}} \tag{24}$$

where $s^{(2)}_{Y_3=k|Y_1=j}$ (and $s^{(2)}_{Y_3=k|Y_1=k}$) refers to the number of grandchildren in position k whose grandparents are in position j (and k).

Note that we define the net and total effects of a prior generation based on a ratio measure, but this tells us nothing about the absolute difference between the number of various types of descendants from two ancestral groups of different social statuses. Revising equation (11), we can

[6] Note that in traditional mobility studies, the mobility effect is often defined in terms of odds ratios, that is $\frac{p_{Y_2=k|Y_1=k}/p_{Y_2=j|Y_1=k}}{p_{Y_2=k|Y_1=j}/p_{Y_2=j|Y_1=j}}$. The odds ratio measure, however, cannot reflect the demography effect if we define SRE as $= \frac{f_k \cdot r_k \cdot p_{Y_2=k|Y_1=k}/f_k \cdot r_k \cdot p_{Y_2=j|Y_1=k}}{f_j \cdot r_j \cdot p_{Y_2=k|Y_1=j}/f_j \cdot r_j \cdot p_{Y_2=j|Y_1=j}}$ because the reproduction parameter as a common factor is cancelled.

[7] In other words, the net social reproduction effect of grandparents can also be defined as $\text{NSRE}^{GP}_{k|j} = \frac{s_{Y_3=k|Y_2=j,Y_1=k}}{s_{Y_3=k|Y_2=j,Y_1=j}} = \frac{f_{jk} \cdot r_{jk} \cdot p_{Y_2=j|Y_1=k}}{f_{jj} \cdot r_{jj} \cdot p_{Y_3=k|Y_2=j,Y_1=j}}$. This NSRE^{GP} will yield the same estimate as that in equation (23) if $\frac{f_{kk}}{f_{kj}} = \frac{f_{jk}}{f_{jj}} = \frac{f_{\cdot k}}{f_{\cdot j}}$, $\frac{r_{kk}}{r_{kj}} = \frac{r_{jk}}{r_{jj}} = \frac{r_{\cdot k}}{r_{\cdot j}}$, and $\frac{p_{Y_3=k|Y_2=k,Y_1=k}}{p_{Y_3=k|Y_2=k,Y_1=j}} = \frac{p_{Y_3=k|Y_2=j,Y_1=k}}{p_{Y_3=k|Y_2=j,Y_1=j}} = \frac{p_{Y_3=k|Y_1=k}}{p_{Y_3=k|Y_1=j}}$.

define SRE in the parent generation as a difference measure:

$$\text{SRE}_{k|j} = s_{Y_2=k|Y_1=k} - s_{Y_2=k|Y_1=j} = f_k \cdot r_k \cdot p_{Y_2=k|Y_1=k} - f_j \cdot r_j \cdot p_{Y_2=k|Y_1=j}. \tag{25}$$

More generally, the net SRE and total SRE of an ancestor who lives n generations back can be written as

$$\text{NSRE}^{(n)}_{k|j} = f_{\bar{\mathbf{Y}}_{n-1,k}} \cdot r_{\bar{\mathbf{Y}}_{n-1,k}} \cdot p_{Y_n=k|\bar{\mathbf{Y}}_{n-1,k}} - f_{\bar{\mathbf{Y}}_{n-1,j}} \cdot r_{\bar{\mathbf{Y}}_{n-1,j}} \cdot p_{Y_n=k|\bar{\mathbf{Y}}_{n-1,j}} \tag{26}$$

$$\text{TSRE}^{(n)}_{k|j} = \sum_{Y_2} \cdots \sum_{Y_{n-1}} f_k \cdot \prod_{i=1}^{n-1} r_{\bar{\mathbf{Y}}_{i,k}} \cdot p_{Y_{i+1}|\bar{\mathbf{Y}}_{i,k}} - f_j \cdot \prod_{i=1}^{n-1} r_{\bar{\mathbf{Y}}_{i,j}} \cdot p_{Y_{i+1}|\bar{\mathbf{Y}}_{i,j}}. \tag{27}$$

The notations are slightly modified from the social reproduction models in equations (13) and (14). In particular, $\bar{\mathbf{Y}}_{i,k}$ refers to $\{Y_i, Y_{i-1} \cdots Y_2, Y_1 = k\}$ in which the social status of the first generation is fixed at $Y_1 = k$. The net effect suggests the comparative advantage of a family with an n^{th} ancestor in social position k over a family with an n^{th} ancestor in position j in producing sons in position k. Social positions of intermediate generations are fixed to be the same for these two families. By contrast, the total effect suggests the cumulative advantage of an ancestor in social position k relative to an ancestor in position j in producing descendants in position k after n generations regardless of social positions of intermediate generations. In equations (26) and (27), the position of the n^{th} generation is fixed at k, namely, $p_{Y_n=k|\bar{\mathbf{Y}}_{n-1,k}}$ (and $p_{Y_n=k|\bar{\mathbf{Y}}_{n-1,j}}$).

4.3 Effect Standardization and Decomposition

Next, we introduce decomposition techniques that separate social reproduction effects into fertility and mobility components. The decomposition method shows the relative importance of demographic and social pathways in explaining differences between descendants from two ancestral groups of different social statuses. To begin, we use Kitagawa's (1955) decomposition method for the difference between two rates. For example, if the number of fathers is standardized (i.e.,

$f_k = f_j = 1$), the difference between the two terms in SRE in equation (25) can be written as

$$\begin{aligned} SRE_{k|j} &= s_{Y_2=k|Y_1=k} - s_{Y_2=k|Y_1=j} \\ &= \underbrace{(r_k - r_j) \cdot \frac{(p_{Y_2=k|Y_1=k} + p_{Y_2=k|Y_1=j})}{2}}_{\text{demography effect}} + \underbrace{\frac{(r_k + r_j)}{2} \cdot (p_{Y_2=k|Y_1=k} - p_{Y_2=k|Y_1=j})}_{\text{mobility effect}} \end{aligned} \tag{28}$$

where the demography effect refers to differences in SRE attributed to differences in reproduction rates of fathers in positions j and k, fixing the mobility probability of their offspring at the level of $\bar{p} = \frac{p_{Y_2=k|Y_1=k}+p_{Y_2=k|Y_1=j}}{2}$; and the mobility effect refers to differences in SRE due to differences in mobility probabilities of offspring from fathers in positions j and k, fixing fathers' reproduction rates at the level of $\bar{r} = \frac{r_k+r_j}{2}$.[8]

If the demographic effect is assumed to have two components—for example, marriage ($m.$) and reproduction within marriage ($r.$), as shown in equation (13)—the above decomposition method can be modified as follows:

$$SRE_{k|j} = (m_k r_k - m_j r_j) \cdot \frac{(p_{Y_2=k|Y_1=k} + p_{Y_2=k|Y_1=j})}{2} + \frac{(m_k r_k + m_j r_j)}{2} \cdot (p_{Y_2=k|Y_1=k} - p_{Y_2=k|Y_1=j}). \tag{29}$$

For the term $(m_k r_k - m_j r_j)$, we can repeat the Kitagawa decomposition method and obtain

$$\begin{aligned} SRE_{k|j} = &\left((m_k - m_j) \cdot \frac{(r_k + r_j)}{2} + (r_k - r_j) \cdot \frac{(m_k + m_j)}{2}\right) \cdot \frac{(p_{Y_2=k|Y_1=k} + p_{Y_2=k|Y_1=j})}{2} \\ &+ \frac{(m_k r_k + m_j r_j)}{2} \cdot (p_{Y_2=k|Y_1=k} - p_{Y_2=k|Y_1=j}). \end{aligned} \tag{30}$$

Let $\bar{m} = \frac{m_k+m_j}{2}$, $\bar{m} = \frac{m_k+m_j}{2}$, $\overline{mr} = \frac{m_k r_k + m_j r_j}{2}$, and $\bar{p} = \frac{p_{Y_2=k|Y_1=k}+p_{Y_2=k|Y_1=j}}{2}$, and the above equation can be further simplified as

$$SRE_{k|j} = \underbrace{(m_k - m_j) \cdot \bar{r} \cdot \bar{p}}_{\text{marriage effect}} + \underbrace{(r_k - r_j) \cdot \bar{m} \cdot \bar{p}}_{\text{reproduction effect}} + \underbrace{\overline{mr} \cdot (p_{Y_2=k|Y_1=k} - p_{Y_2=k|Y_1=j})}_{\text{mobility effect}}. \tag{31}$$

There are many ways to decompose a difference measure, especially when the demographic rate contains multiple factors. For example, the above SRE with marriage, reproduction, and mobility

[8] Following the definitions in equation (23), the difference measure of the net social reproduction effect of grandparents can be decomposed into $\text{NSRE}^{GP}_{k|j} = (r_{kk} - r_{kj}) \cdot \frac{(p_{Y_3=k|Y_2=k,Y_1=k}+p_{Y_3=k|Y_2=k,Y_1=j})}{2} + \frac{(r_{kk}+r_{kj})}{2} \cdot (p_{Y_3=k|Y_2=k,Y_1=k} - p_{Y_3=k|Y_2=k,Y_1=j})$.

components can also be decomposed using the method proposed by Das Gupta (1993):

$$\text{marriage effect} = \left[\frac{r_k \cdot p_{Y_2=k|Y_1=k} + r_j \cdot p_{Y_2=k|Y_1=j}}{3} + \frac{r_k \cdot p_{Y_2=k|Y_1=j} + r_j \cdot p_{Y_2=k|Y_1=k}}{6}\right] \cdot (m_k - m_j) \tag{32}$$

$$\text{reproduction effect} = \left[\frac{m_k \cdot p_{Y_2=k|Y_1=k} + m_j \cdot p_{Y_2=k|Y_1=j}}{3} + \frac{m_k \cdot p_{Y_2=k|Y_1=j} + m_j \cdot p_{Y_2=k|Y_1=k}}{6}\right] \cdot (r_k - r_j) \tag{33}$$

$$\text{mobility effect} = \left[\frac{m_k \cdot r_k + m_j \cdot r_j}{3} + \frac{m_k \cdot r_j + m_j \cdot r_k}{6}\right] \cdot (p_{Y_2=k|Y_1=k} - p_{Y_2=k|Y_1=j}) \tag{34}$$

Next, we extend equation (28) and consider the standardized difference measure of the total effect of grandparents. For the sake of simplicity, we omit the marriage effect. A simple decomposition method that divides the effect into total demographic effects and total mobility effects would be

$$\text{TSRE}^{GP}_{k|j} = \sum_i r_k \cdot p_{Y_2=i|Y_1=k} \cdot r_{ik} \cdot p_{Y_3=k|Y_2=i,Y_1=k} - \sum_i r_j \cdot p_{Y_2=i|Y_1=j} \cdot r_{ij} \cdot p_{Y_3=k|Y_2=i,Y_1=j} \tag{35}$$

$$= \underbrace{\sum_i (r_k \cdot r_{ik} - r_j \cdot r_{ij}) \cdot \frac{(p_{Y_2=i|Y_1=k} \cdot p_{Y_3=k|Y_2=i,Y_1=k} + p_{Y_2=i|Y_1=j} \cdot p_{Y_3=k|Y_2=i,Y_1=j})}{2}}_{\text{demography effect}}$$

$$+ \underbrace{\sum_i \frac{(r_k \cdot r_{ik} + r_j \cdot r_{ij})}{2} \cdot (p_{Y_2=i|Y_1=k} \cdot p_{Y_3=k|Y_2=i,Y_1=k} - p_{Y_2=i|Y_1=j} \cdot p_{Y_3=k|Y_2=i,Y_1=j})}_{\text{mobility effect}} \tag{36}$$

Applying Das Gupta's decomposition method (1993) for rates of four factors, we can further decompose compound demography and mobility effects into effects from different generations. To simplify the notations, below we use $r_1 = r_k$, $r'_1 = r_j$, $r_2 = r_{ik}$, $r'_2 = r_{ij}$ $p_1 = p_{Y_2=i|Y_1=k}$, $p'_1 = p_{Y_2=i|Y_1=j}$, $p_2 = p_{Y_3=k|Y_2=i,Y_1=k}$, $p'_2 = p_{Y_3=k|Y_2=i,Y_1=j}$.

$$\text{demography effect (1)} = \sum_{Y_2}\left[\frac{p_1 r_2 p_2 + p'_1 r'_2 p'_2}{4} + \frac{p_1 r_2 p'_2 + p_1 r'_2 p_2 + p'_1 r_2 p_2 + p'_1 r'_2 p_2 + p'_1 r_2 p'_2 + p_1 r'_2 p'_2}{12}\right]$$
$$\cdot (r_1 - r'_1) \tag{37}$$

$$\text{mobility effect (1)} = \sum_{Y_2} \left[\frac{r_1 r_2 p_2 + r_1' r_2' p_2'}{4} + \frac{r_1 r_2 p_2' + r_1 r_2' p_2 + r_1' r_2 p_2 + r_1' r_2' p_2 + r_1' r_2 p_2' + r_1 r_2' p_2'}{12} \right] \cdot (p_1 - p_1') \tag{38}$$

$$\text{demography effect (2)} = \sum_{Y_2} \left[\frac{p_1 r_1 p_2 + p_1' r_1' p_2'}{4} + \frac{p_1 r_1 p_2' + p_1 r_1' p_2 + p_1' r_1 p_2 + p_1' r_1' p_2 + p_1' r_1 p_2' + p_1 r_1' p_2'}{12} \right] \cdot (r_2 - r_2') \tag{39}$$

$$\text{mobility effect (2)} = \sum_{Y_2} \left[\frac{p_1 r_1 r_2 + p_1' r_1' r_2'}{4} + \frac{p_1 r_1 r_2' + p_1 r_1' r_2 + p_1' r_1 r_2 + p_1' r_1' r_2 + p_1' r_1 r_2' + p_1 r_1' r_2'}{12} \right] \cdot (p_2 - p_2') \tag{40}$$

Similarly, to derive the demography and mobility effects from the total effect of great-grandparents, we can apply Das Gupta's method for rates of six factors. For example, the demography effect from the first generation r_1 versus r_1' is:

$$\begin{aligned} \text{demography effect (1)} = \sum_{Y_2} \sum_{Y_3} \Bigg[& \frac{p_1 r_2 p_2 r_3 p_3 + p_1' r_2' p_2' r_3' p_3'}{6} \\ & + \frac{\begin{array}{c} p_1 r_2 p_2 r_3 p_3' + p_1 r_2 p_2 r_3' p_3 + p_1 r_2 p_2' r_3 p_3 + p_1 r_2' p_2 r_3 p_3 + p_1' r_2 p_2 r_3 p_3 \\ + p_1' r_2' p_2' r_3' p_3 + p_1' r_2' p_2' r_3 p_3' + p_1' r_2' p_2 r_3' p_3' + p_1' r_2 p_2' r_3' p_3' + p_1 r_2' p_2' r_3' p_3' \end{array}}{30} \\ & + \frac{\begin{array}{c} p_1 r_2 p_2 r_3' p_3' + p_1 r_2 p_2' r_3 p_3' + p_1 r_2 p_2' r_3' p_3 + p_1 r_2' p_2 r_3 p_3' + p_1 r_2' p_2 r_3' p_3 \\ + p_1 r_2' p_2' r_3 p_3 + p_1' r_2 p_2 r_3 p_3' + p_1' r_2 p_2 r_3' p_3 + p_1' r_2 p_2' r_3 p_3 + p_1' r_2' p_2 r_3 p_3 \\ + p_1' r_2' p_2' r_3 p_3 + p_1' r_2' p_2 r_3' p_3 + p_1' r_2' p_2 r_3 p_3' + p_1' r_2 p_2' r_3' p_3 + p_1' r_2 p_2' r_3 p_3' \\ + p_1' r_2 p_2 r_3' p_3' + p_1 r_2' p_2' r_3' p_3 + p_1 r_2' p_2' r_3 p_3' + p_1 r_2' p_2 r_3' p_3' + p_1 r_2 p_2' r_3' p_3' \end{array}}{60} \Bigg] \\ & \cdot (r_1 - r_1') \end{aligned} \tag{41}$$

Demography effects (2)–(3) and mobility effects (1)–(3) can be derived easily by interchanging the terms in equation (41). The total effect of great-grandparents is equal to the sum of all these separate effects.

In general, the total effect of an N^{th} ancestor defined in equation (27) can be decomposed into $2N$ terms including demographic effects and mobility effects from each of the N prior generations. Below we apply the decomposition method of rates as the product of P factors proposed by Das Gupta (1993). We first simplify the notations for demographic and mobility parameters in each generation. The demographic parameter in the n^{th} generation ($n = 1 \cdots N$) is written as

$$r_n = r_{Y_n|\bar{\mathbf{Y}}_{n-1,k}} \text{ and } r'_n = r_{Y_n|\bar{\mathbf{Y}}_{n-1,j}}. \tag{42}$$

We use r and r' to differentiate between two ancestors in the founding generation with social status k and j, respectively. Similarly, the mobility parameters in the n^{th} generation are

$$p_n = p_{Y_{n+1}|\bar{\mathbf{Y}}_{n,k}} \text{ and } p'_n = p_{Y_{n+1}|\bar{\mathbf{Y}}_{n,j}}. \tag{43}$$

Suppose the elements of r and p are members of set $\mathbb{A} = \{r_1, \cdots, r_N, p_1, \cdots, p_N\}$ and r' and p' are members of set $\mathbb{A}'$. The set $\mathbb{A}$ excluding one element A_n (e.g., r_n) is defined as $\mathbb{A}\backslash A_n$ (or $\mathbb{A}\backslash r_n$). The $\text{TSRE}^{(n)}_{k|j} = \sum_{Y_2} \cdots \sum_{Y_{n-1}} r_1 \cdots r_{N-1} \cdot p_1 \cdots p_{N-1} - r'_1 \cdots r'_{N-1} \cdot p'_1 \cdots p'_{N-1}$ can be decomposed into the sum of demography effect (n) and mobility effect (n) from the n^{th} generation. For example, Das Gupta (1993: 15–16) describes the decomposition for the part $r_1 \cdots r_{N-1} \cdot p_1 \cdots p_{N-1} - r'_1 \cdots r'_{N-1} \cdot p'_1 \cdots p'_{N-1}$ as

$$\text{demography effect } (n) = \sum_{t=1}^{N} \frac{\begin{array}{c}\text{sum of all } (2N-1) \text{ terms with } (2N-t) \text{ from set } \mathbb{A}\backslash r_n \text{ and } (t-1) \\ \text{from set } \mathbb{A}'\backslash r'_n \text{ or } (2N-t) \text{ terms from } \mathbb{A}'\backslash r'_n \text{ and } (t-1) \text{ from } \mathbb{A}\backslash r_n\end{array}}{2N \cdot \binom{2N-1}{t-1}} \cdot (r_n - r'_n) \tag{44}$$

More formally, we introduce the following notations to define the demography effect in equation (44). We denote $\mathbb{B}_{2N-t}$ as subsets of $\mathbb{A}\backslash A_n$ with a cardinality of $2N-t$ (i.e., $|\mathbb{B}| = 2N-t$). Given that there are $\binom{2N-1}{2N-t}$ of such subsets, each subset i is denoted by

$$\mathbb{B}_{2N-t,i} = \{B_{2N-t,i} : B_{2N-t,i} \in \mathbb{A}\backslash A_n\}.$$

The complement of set $\mathbb{B}_{2N-t,i}$ can be written as $\bar{\mathbb{B}}_{2N-t,i}$, which satisfies that $\bar{\mathbb{B}}_{2N-t,i} = \mathbb{B}_{t-1,i}$ with the cardinality of $t-1$. Taking our illustration of the total effect of grandparents with $N = 2$ as an example, the set $\mathbb{B}_{21} = \{r_2, p_2\}$ is one subset with cardinality 2 of the set $\mathbb{A}\backslash r_1 = \{r_2, p_1, p_2\}$. Other subsets include $\mathbb{B}_{22} = \{r_2, p_1\}$ and $\mathbb{B}_{23} = \{p_1, p_2\}$, where the total number of subsets with cardinality 2 is $\binom{3}{2} = 3$. The complement set of $\mathbb{B}_{21}$ in the counterpart set of $\mathbb{A}'$ is $\bar{\mathbb{B}}'_{21} = \mathbb{B}'_{11} = \{p'_1\}$.

$$\text{demography effect } (n) = \frac{\sum_{t=1}^{N} \sum_{i=1}^{\binom{2N-1}{2N-t}} \left(\prod_{B \in \mathbb{B}_{2N-t}} B_{2N-t,i} \cdot \prod_{B' \in \mathbb{B}'_{t-1}} B'_{t-1,i} + \prod_{B' \in \mathbb{B}'_{2N-t}} B'_{2N-t,i} \cdot \prod_{B \in \mathbb{B}_{t-1}} B_{t-1,i} \right)}{2N \cdot \binom{2N-1}{t-1}} \cdot (r_n - r'_n) \tag{45}$$

Likewise, if we define set $\mathbb{B}$ as a subset of $\mathbb{A} \backslash p_n$, the mobility effect can be written as

$$\text{mobility effect } (n) = \frac{\sum_{t=1}^{N} \sum_{i=1}^{\binom{2N-1}{2N-t}} \left(\prod_{B \in \mathbb{B}_{2N-t}} B_{2N-t,i} \cdot \prod_{B' \in \mathbb{B}'_{t-1}} B'_{t-1,i} + \prod_{B' \in \mathbb{B}'_{2N-t}} B'_{2N-t,i} \cdot \prod_{B \in \mathbb{B}_{t-1}} B_{t-1,i} \right)}{2N \cdot \binom{2N-1}{t-1}} \cdot (p_n - p'_n) \tag{46}$$

5 SHORT-TERM EFFECT, LONG-TERM EFFECT, AND EQUILIBRIUM EFFECT

Based on the recursive form of the multigenerational model in equation (8), we can quantify the relative importance of mobility and demography in shaping short-term and long-term family inequality dynamics. We can also simulate the equilibrium properties of the model regarding the eventual population composition of individuals descended from different families. A critical property of the Markov model is that if the transition matrix is time-invariant, the distribution of social classes after t generations, that is, $\mathbf{S}^{(t)} = \mathbf{F}^{(0)}\mathbf{P}^{(t)}$, will gradually converge to a stationary distribution that is only determined by the transition matrix, not by the initial distribution of $\mathbf{F}^{(0)}$.[9] This property suggests a family's initial social class may influence the social mobility probability for several generations, but eventually its influence will fade away. As such, after enough generations, the social class of a descendant will eventually become independent of the social class of the lineage founder. The social class distribution between descendants from high-status and low-status origin lineage founders will become identical. Therefore, social advantages associated with any generation will not permanently change the social prospects of future generations. Any short-term effect of

[9]This conclusion relies on the assumption that $p_{ij} > 0$, so that the Markov chain is aperiodic and has a single recurrent class. This condition ensures that the Markov chain of the mobility process can always converge to a stable distribution ("equilibrium"), which is independent of the initial distribution.

one generation does not translate into long-term inequality between families that originate from unequal social statuses. This property is illustrated by equations (47) and (48).

First, let us define the long-term mobility effect as the ratio of descendants in class k from a lineage founder in class k relative to class j:

$$\text{LSRE} = \lim_{t\to\infty} \left(\frac{\mathbf{S}_{k|k}^{(t)}}{\mathbf{S}_{k|j}^{(t)}} \right). \tag{47}$$

Using the property defined in equation (10), we obtain,

$$\text{LSER} = \lim_{t\to\infty} \left(\frac{\mathbf{F}_k^{(0)} \cdot \mathbf{P}^{(t)}}{\mathbf{F}_j^{(0)} \cdot \mathbf{P}^{(t)}} \right) = \frac{\pi}{\pi} = 1. \tag{48}$$

The LSRE will also converge to 1 in the long run if social mobility follows a second- or higher-order Markov process with a time-invariant transition matrix. Such a conclusion, however, may not hold when we examine the social reproduction effect rather than the mobility effect alone. Assume $\mathbf{S}^{(t)} = \mathbf{F}^{(0)} \cdot \mathbf{C}^{(t)}$, where $\mathbf{C} = \mathbf{R} \cdot \mathbf{P}$, a combination of the reproduction and mobility components. Suppose the number of social classes is the same for fathers and sons, and the marriage, fertility, and mobility matrices have no structural 0s. That is, men in different social classes may get married and have sons, and all sons may stay in the same social class as their fathers or move to other classes. According to the Perron-Frobenius theorem, $\mathbf{C}$ would be a square matrix with positive entries and a unique dominant eigenvalue. The long-term behavior of $\mathbf{S}^{(t)}$ would depend on the largest eigenvalue of $\mathbf{C}$.

To see this, we assume $\mathbf{C}$ has n linearly independent left eigenvectors $\mathbf{v}_1, \mathbf{v}_2 \ldots \mathbf{v}_n$ with corresponding eigenvalues of $\lambda_1, \lambda_2, \ldots \lambda_n$. Assume the eigenvalues are ordered so that $|\lambda_1| > \cdots \geq |\lambda_{n-1}| \geq |\lambda_n|$. For the social class distribution in the first generation $\mathbf{S}^{(1)}$, we can write this vector as the linear combination of the eigenvectors of $\mathbf{C}$:

$$\mathbf{S}^{(1)} = a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + \cdots + a_n\mathbf{v}_n \tag{49}$$

where $a_1 \cdots a_n$ are scalars and $a_1 \neq 0$. Then, multiplying both sides by $\mathbf{C}$ produces

$$\mathbf{S}^{(1)} \cdot \mathbf{C} = (a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + \cdots + a_n\mathbf{v}_n) \cdot \mathbf{C} \tag{50}$$

Using the spectral decomposition theorem,

$$\mathbf{S}^{(1)} \cdot \mathbf{C} = a_1(\lambda_1 \mathbf{v}_1) + a_2(\lambda_2 \mathbf{v}_2) + \cdots + a_n(\lambda_n \mathbf{v}_n). \tag{51}$$

Repeating the multiplication on both sides produces

$$\mathbf{S}^{(1)} \cdot \mathbf{C}^{(t-1)} = a_1(\lambda_1^{t-1} \mathbf{v}_1) + a_2(\lambda_2^{t-1} \mathbf{v}_2) + \cdots + a_n(\lambda_n^{t-1} \mathbf{v}_t) = \mathbf{S}^{(t)}. \tag{52}$$

As λ_1 is assumed to be larger in absolute value than the other eigenvalues, it follows that each of the fractions $\frac{\lambda_2}{\lambda_1}, \frac{\lambda_3}{\lambda_1} \ldots \frac{\lambda_n}{\lambda_1}$ is less than 1 in absolute value. Each of the factors $\left(\frac{\lambda_2}{\lambda_1}\right)^{t-1}, \left(\frac{\lambda_3}{\lambda_1}\right)^{t-1} \ldots \left(\frac{\lambda_n}{\lambda_1}\right)^{t-1}$ must converge to 0 as $t-1$ approaches infinity. Therefore, we have the relationship

$$\mathbf{S}^{(t)} \simeq a_1 \left(\lambda_1^{t-1} \mathbf{v}_1\right). \tag{53}$$

For the initial vector $\mathbf{F}^{(0)} = [f_1, f_2, \cdots, f_n]$, let $\mathbf{F}_k^{(0)} = [0, \cdots f_k = 1, \cdots 0]$ and $\mathbf{F}_j^{(0)} = [0, \cdots f_j = 1, \cdots 0]$. In other words, the entire initial cohort was located in a single class. Assume $a_1 = a_{1k}$, when $\mathbf{S}^{(1)} = \mathbf{F}_k^{(0)}\mathbf{C}$, and $a_1 = a_{1j}$, when $\mathbf{S}^{(1)} = \mathbf{F}_j^{(0)}\mathbf{C}$. After t generations, the long-term social reproduction effect would converge to

$$\text{LSRE} = \lim_{t \to \infty} \left(\frac{a_{1k} \lambda_1^{t-1} \mathbf{v}_1}{a_{1j} \lambda_1^{t-1} \mathbf{v}_1} \right) = \frac{a_{1k}}{a_{1j}}. \tag{54}$$

After enough generations, families starting with social class k would eventually produce $\frac{a_{1k}}{a_{1j}}$ times as many descendants in class k as families starting with social class j do. In other words, the asymptotic distribution of descendants is path-dependent: not all families produce the same number of descendants in the long run. By contrast, regular Markov mobility models with no demographic effects are ergodic. The transition matrix $\mathbf{P}$ can be viewed as a special case of matrix $\mathbf{C}$, where matrix $\mathbf{C}$ is subject to the constraint that the sum of each row is 1. This constraint also guarantees that $\lambda_1 = 1$ and $a_{1k} = a_{1j}$. As a result, the equilibrium distribution of the Markov mobility model will not depend on social class of the initial generation.

6 HETEROGENEOUS MOBILITY REGIMES

The regular Markov mobility model assumes all individuals in a population have identical transition probabilities conditional on their parents' social status (**Assumption 4**). This assumption overlooks many possible sources of heterogeneity associated with individual-level social attributes and macro-level social structures (Blau 1977). Below, I group sources of population heterogeneity discussed in previous mobility research into three types: (1) individual, time-invariant heterogeneity, (2) individual, time-dependent heterogeneity, and (3) heterogeneity in mobility regimes.

The idea of mobility heterogeneity with time-invariant properties can be traced back to Blumen et al.'s (1955) pioneering study on intragenerational labor mobility. A similar conception can be applied to the analysis of intergenerational mobility of family lines. Blumen et al. (1955) identify two types of individuals in a population: movers, who change jobs over time according to a time-constant Markov transition matrix, and stayers, who remain in the same job category with probability 1 (namely, a diagonal matrix for the transition matrix). The model includes the proportions of movers and stayers and the transition probability matrix for movers. A person's attribute—as either a mover or a stayer—does not change during the entire period of study. The model is formulated as follows:

$$\mathbf{S} = \mathbf{F} \cdot (\mathbf{\Lambda} + (\mathbf{I} - \mathbf{\Lambda}) \cdot \mathbf{P}) \tag{55}$$

where $\mathbf{S}$ and $\mathbf{F}$ are social class distributions of sons and fathers, respectively; $\mathbf{\Lambda}$ is a diagonal matrix with the proportions of stayers in each social class on the diagonals; the diagonal matrix $\mathbf{I} - \mathbf{\Lambda}$ includes the proportions of movers as diagonal entries; and the matrix $\mathbf{P}$ refers to the transition mobility matrix for the movers. The mover-stayer model considers only one of many possible types of time-invariant heterogeneity by assuming only two distinct subpopulations. In general, individuals may conform to miscellaneous transition probabilities, or in a more extreme scenario, each person follows a mobility process governed by a unique set of mobility probabilities (Xie 2013).[10]

To model individual heterogeneity, Spilerman (1972a) proposes a Markov model that estimates

[10]Previous research on continuous Markov chain models also discusses heterogeneity in mobility rates versus heterogeneity in transition matrices (Bartholomew 1967; Spilerman 1972b).

individual transition probability with regression models. We first convert the father-son transition matrix into a person-transition dataset (a long format), where each possible transition is represented as its own observation.[11] For each father in social class i and son in social class j, define an indicator variable $Z_{ij} = \{z_{Y_1=i,Y_2=1}, z_{Y_1=i,Y_2=2}, \cdots, z_{Y_1=i,Y_2=J}\}$

$$Z_{ij} = \begin{cases} 1 & \text{if } Y_2 = j \mid Y_1 = i \\ 0 & \text{if } Y_2 \neq j \mid Y_1 = i \end{cases} \tag{56}$$

In other words, the indicator variable is 1 if a person born into class i moves to class j. The variable is 0 for other counterfactual scenarios. We then fit $I \times J$ (often $I = J$) linear probability equations, and in each equation, Z_{ij} is predicted by a set of social attributes X.

For individual c from social class i with attributes $(X_{1c}, X_{2c}, \cdots, X_{Vc})$, his probability of moving into class j is

$$\Pr(Z_{ijc} \mid X_c) = a_{ij} + \sum_{v=1}^{V} b_{ijv} X_{vc} \tag{57}$$

where $\hat{a}_{ij}$ and $\hat{b}_{ijv}$ are regression coefficient estimates. We can also use other regression models, such as logistic or probit functions, to estimate the transition probability. If we estimate a separate transition matrix $\mathbf{P}_c(X_c)$ for each person c using the predicted probabilities based on attributes X_c, intergenerational mobility from fathers to sons can be represented in the following matrix form:

$$\mathbf{S} = \mathbf{F} \cdot \left(C^{-1} \cdot \sum_{c=1}^{C} \mathbf{P}_c(X_c) \right) \tag{58}$$

where $\mathbf{P}(X_{vc})$ is known as the individual transition matrix (McFarland 1970; Spilerman 1972a). The overall transition matrix is estimated from the sum of all individual transition matrices divided by the population size C.[12]

The second type of heterogeneity assumes the mobility matrix operates as a function of time. Studies on intragenerational mobility have proposed the "Retention Model" (Henry 1971) and the

[11] Assume we have N father-son dyads in the data. To generate the indicator variable, we create J observations for each son and the sample size becomes $N \times J$.

[12] If we are only interested in the intergenerational mobility of a certain social group (e.g., non-white, N_c), the intergenerational mobility can be represented as: $\mathbf{S} = \mathbf{F} \cdot \left(N_C^{-1} \cdot \sum_{c=1}^{N_C} \mathbf{P}_c(X_c) \right)$.

"Cornell Mobility Model" (McGinnis 1968), in which movers' transition probability is assumed to change over time. Below, we apply these models to the analysis of intergenerational mobility. We define stayers as individuals who remain in the same occupation as their parents and movers as those who enter an occupation different from that of their parents. These models show examples when the stationarity assumption (**Assumption 3**) fails.

The Retention Model modifies the mover-stayer model in equation (55) by allowing the proportion of movers and stayers to be time-dependent, so the mover-stayer model becomes

$$\mathbf{S} = \mathbf{F} \cdot (\mathbf{\Lambda_t} + (\mathbf{I} - \mathbf{\Lambda_t}) \cdot \mathbf{P}) \tag{59}$$

where $\mathbf{\Lambda_t}$ is a diagonal matrix with the diagonal cells indicating the proportion of stayers in each social class as a function of time; and $\mathbf{P}$ refers to the transition matrix of the movers.

The Cornell Mobility model postulates that individuals' tendency to leave a social class declines as a strictly monotonic function of the duration of staying in that class. From an intergenerational perspective, this "cumulative inertia" property implies that the longer a family has been in its current social class, the higher its probability of remaining there for yet another generation. Following the notation in McGinnis (1968), let ${}_d\mathbf{P}_{Y_2=j|Y_i}(t)$ refer to the transition probability from class i to class j at generation t when a family has remained in class i for d consecutive generations prior to generation t. The distribution of social classes in the father's generation is $\mathbf{F} = [{}_1f_1, {}_1f_2, \cdots, {}_1f_I, {}_2f_1, {}_2f_2, \cdots, {}_df_1, \cdots, {}_df_I]$. The duration-specific transition matrix can be partitioned into the transition matrices of movers and stayers, both of which are also duration specific:

$${}_d\mathbf{P}_{Y_2=j|Y_1=i}(t) = {}_d\mathbf{P}^{\text{mover}}_{Y_2=j|Y_1=i}(t) + {}_d\mathbf{P}^{\text{stayer}}_{Y_2=j|Y_1=i}(t) \tag{60}$$

The stayers' transition matrix ${}_d\mathbf{P}^{\text{stayer}}_{Y_2=j|Y_1=i}(t)$ is the diagonalization of ${}_d\mathbf{P}_{Y_2=j|Y_1=i}(t)$, which satisfies that

$${}_d\mathbf{P}^{\text{stayer}}_{Y_2=j|Y_1=i}(t) = \begin{cases} {}_d\mathbf{P}_{Y_2=j|Y_1=i}(t) & \text{if } j = i \\ 0 & \text{otherwise} \end{cases} \tag{61}$$

The relationship between the movers' and stayers' transition matrices satisfies that

$$_{d}\mathbf{P}^{\text{mover}}_{Y_2=j|Y_1=i}(t) = (\mathbf{I} - {}_{d}\mathbf{P}^{\text{stayer}}_{Y_2=j|Y_1=i}(t)) \cdot \mathbf{R} \tag{62}$$

where $\mathbf{R} = [r_{ij}]$ subject to $r_{ii} = 0$ is shown to always exist and does not vary by d. Because of the cumulative inertia property, the model also requires that $_{d+1}\mathbf{P}^{\text{stayer}}_{Y_2=j|Y_1=i} > {}_{d}\mathbf{P}^{\text{stayer}}_{Y_2=j|Y_1=i}$.[13]

Note that this model shows a violation of the stationarity assumption in the regular Markov model by introducing a time component into the transition probability. Also, it violates the Markovian assumption by linking individuals' mobility outcomes to the entire history of moves in previous generations.

The third type of heterogeneity concerns the mixture of mobility regimes in a society or in a broader stratification system created by spatial, cultural, or institutional forces of segregation. Using occupational mobility as an example, a plethora of studies show variation in intergenerational mobility among industrial societies in the mid to late 20th century (DiPrete 2002; Lipset and Bendix 1959; Featherman et al. 1975; Grusky and Hauser 1984; Erikson and Goldthorpe 1992; Xie 1992; Yamaguchi 1987). Even within a society, several mobility regimes may coexist. One instructive example provided by Mare (2011) postulates a stratification system consisting of three strata: the top class, the middle and working class, and the bottom class. The persistence of social status is stronger at the top and bottom of the social hierarchy than in the middle, due to social policies and family norms that create a cumulative advantage or disadvantage for families. In cross-country comparisons, social boundaries among mobility regimes are often assumed to be impermeable, in contrast to within-society comparisons in which families that start in one mobility regime may circulate in and out of other regimes after generations of movement. These mobility processes can be represented in the following matrix form:

$$\mathbf{S}_{1\times n} = \mathbf{F}_{1\times n} \cdot \left(\sum_{c=1}^{C} \lambda(c) \cdot \mathbf{P}_{n\times n}(c) \right) \tag{63}$$

where $\lambda(c)$ denotes the weight of each subgroup c, $\lambda(c) > 0$, $\sum_c \lambda(c) = 1$ and $p_{ij}(c)$ denotes the mobility probability of sons achieving social status j given their fathers are in class i for subgroup c

[13] One example illustrated by McGinnis (1968) is $_{d}\mathbf{P}^{\text{stayer}} = \mathbf{I} - \left(1 - \frac{1}{a}\right)^{d-1} \left(\mathbf{I} - {}_{1}\mathbf{P}^{\text{stayer}}\right)$, $a > 1$ and $_{d}\mathbf{P}^{\text{mover}} = \left(1 - \frac{1}{a}\right)^{d-1} (\mathbf{I} - {}_{d}\mathbf{P}^{\text{stayer}}) \cdot \mathbf{R}$.

($c = 1, 2, ..., C$). For example, Mare and Song (2014) analyzed social mobility of descendants from imperial and peasant families using Chinese family genealogies and historical linked censuses ($c =$ 1 descendants of emperors; $c = 2$ descendants of peasants). If subgroups are distinct and time-invariant, then $p_{ij}(c)$ will be fixed over generations. If social boundaries are permeable, families' membership c may change over time, leading to time-varying weights $\lambda(c, t)$ and mobility matrix $\mathbf{P}(c, t)$.

Compared to regular mobility models, mixture Markov models that account for population heterogeneity often provide better representations of observed mobility processes, in terms of model goodness-of-fit such as the χ^2 test (Goodman 1962). However, as McFarland (1970: 475) notes regarding these heterogeneous mobility models, "any real adequate model would be too cumbersome to be fitted to numerical data." Mobility researchers will always have to design a trade-off between model accuracy and simplicity.

7 A TWO-SEX APPROACH

Traditional social mobility studies typically take a one-sex approach by focusing exclusively on the intergenerational association of social status between fathers and sons, while ignoring the independent role of mothers or the joint role of parents (**Assumption 5**). The one-sex approach is also widely adopted in demographic models, which assume population dynamics are determined by the vital rates of one sex only, often women, or that the roles of both sexes are identical (Caswell 2001). In multigenerational analyses, the one-sex approach is useful when transmission of social characteristics is sex-linked. For example, in many patriarchal societies, social positions were passed down the male lineage from paternal grandfathers to fathers to sons. When comparing descendants who carry sex-linked characteristics from families of unequal origins, we only need to count male descendants in the population. In most western societies, however, this theoretical assumption may be invalid in practice (Song and Mare 2017).

First, mothers, grandmothers, and maternal grandparents may influence individuals' socioeconomic outcomes, in addition to the influences of patrilineal kin. Beller (2009) shows that the occupational association between mothers and offspring is approximately equal to that of fathers and offspring, partly due to the recent rise in female labor force participation, the reversed gender gap in higher education, and increasing gender economic equality. Also, maternal families, especially maternal grandmothers may be particularly important for some subpopulations, such as single-parent or skipped-generation households, where grandparents help raise their grandchildren.

Second, mobility probabilities and demographic rates may differ by gender for sociological and biogenetic reasons. Men and women often face unequal opportunity structures because of their distinct occupational preferences, human capital differences that develop through education and work experiences, and career trajectories (Reskin 1993; Jacobs 1989). Compared to their female counterparts, men often have higher mortality rates in childhood and adulthood (Preston et al. 2001), lower marriage rates, and greater variation in fertility rates.[14] Additionally, socioeconomic gradients in demographic rates also vary by gender. For example, Nettle and Pollet (2008) show that income is positively associated with fertility for men but negatively for women. If we apply the one-sex social and demographic mobility model in equation (13) for men and women separately, the results may disagree.

Third, the one-sex approach does not take into account interactions between men and women through the formation of marriages, a mechanism that determines the social makeup of families and creates the family background of the next generation (Mare and Schwartz 2006). The formation of marriages depends on the abundance and preferences of mates in a population. On the one hand, the number and types of marriages are constrained by the "marriage squeeze," or the imbalance between the number of men and women considered marriageable (Akers 1967; Schoen 1983). The marriage squeeze may produce significant changes in the timing and patterns of marriage and fertility levels in a population. On the other hand, men and women tend to marry others with similar socioeconomic characteristics. The degree of assortative mating influences not only socioeconomic resemblance between couples, but also the amount of social advantage or disadvantage transmitted to the offspring generation. Compared to the one-sex approach, the two-sex approach provides a refined picture of how intermarriages among social classes, in combination with differential fertility and mortality, shape social inequality among families.

Previous demographic studies have proposed a variety of two-sex models that account for the

[14] http://www.pewsocialtrends.org/2014/09/24/record-share-of-americans-have-never-married/

interdependence of demographic behaviors of both sexes in determining the number of marriages and births in a population. These models have been used to predict the size, composition, and growth of future populations (Caswell 2001; Caswell and Weeks 1986; Goodman 1953, 1968; Jenouvrier et al. 2010; Kendall 1949; Miller and Inouye 2011; Pollak 1986; Pollard 1973). Below, I illustrate how to adapt these demographic models for multigenerational mobility research, using the *Birth Matrix-Mating Rule* (BMMR) model developed by Pollak (1986, 1987, 1990a,b) as an example. In parallel to the one-sex model in equation (8), the two-sex model for men and women is specified as

$$s_k = \sum_i \sum_j \mu_{ij}(\mathbf{N}^m, \mathbf{N}^f) \cdot r_{ij}^m \cdot p^m_{Y_2=k|Y_1=\{i,j\}} \tag{64}$$

$$d_k = \sum_i \sum_j \mu_{ij}(\mathbf{N}^m, \mathbf{N}^f) \cdot r_{ij}^f \cdot p^f_{Y_2=k|Y_1=\{i,j\}} \tag{65}$$

where s_k (d_k) denotes the number of sons (daughters) in the offspring generation who are in social class k; $\mu_{ij}(\mathbf{N}^m, \mathbf{N}^f)$ denotes the number of marriages between fathers in class i and mothers in class j;[15] and r_{ij}^m (r_{ij}^f) denotes the mean number of surviving sons (or daughters) born from each union of class i fathers and class j mothers with completed reproduction history. In general, the differences between r_{ij}^m and r_{ij}^f are determined by male-to-female sex ratios at birth in a population and differential survival rates of boys and girls to adulthood. In most populations, the two estimates can be considered equal. Finally, $p^m_{Y_2=k|Y_1=\{i,j\}}$ and $p^f_{Y_2=k|Y_1=\{i,j\}}$ refer to the probability of obtaining class k for sons and daughters born to class i fathers and class j mothers, respectively.

To model the mating rule term, $\mu_{ij}(\mathbf{N}^m, \mathbf{N}^f)$, I adopt Schoen's harmonic mean mating rule (Schoen 1981, 1988), which assumes the number of marriages between two social groups depends on the relative number of single women and men in these groups and the attractiveness of these group members to each other. The harmonic mean mating rule specifies that

$$\mu_{ij}(\mathbf{N}^m, \mathbf{N}^f) = \frac{\alpha_{ij}\mathbf{N}_i^m \mathbf{N}_j^f}{\mathbf{N}_i^m + \mathbf{N}_j^f}, \quad \alpha_{ij} > 0, \sum_j \alpha_{ij} \leq 1, \sum_i \alpha_{ij} \leq 1 \tag{66}$$

[15] Following the tradition in the demographic literature, the number of marriages between fathers in class i and mothers in class j is denoted as $\mu_{ij}(\mathbf{N}^m, \mathbf{N}^f)$ rather than $\mu(\mathbf{N}_i^m, \mathbf{N}_i^f)$, because the number of marriages between men in class i and women in class j may depend on the number of men and women in social classes other than i and j, namely, competition among different classes.

where α_{ij} is the "force of attraction" between males in class i and females in class j, which results from constraints imposed by the abundance of mates as well as preferences among all class groups (Schoen 1988). In empirical studies, α_{ij} is often estimated from the number of marriages and single individuals in different social classes (Qian 1998; Qian and Preston 1993; Raymo and Iwasawa 2005). $\mathbf{N}_i^m$ is the total number of eligible men in class i, and $\mathbf{N}_j^f$ is the total number of eligible women in class j. One limitation of this function is that it assumes no competition among different classes ("zero spillover mating rule") (Pollak 1990a). Miller and Inouye (2011) provide a list of candidate two-sex mating rules and evaluate their pros and cons using empirical data.

Using the two-generation model in equations (64) and (65), we can derive the socioeconomic distribution of the grandchild generation. Specifically, the number of granddaughters (grandsons) in class k, denoted as $d_k^{(2)}$ ($s_k^{(2)}$), can be estimated as:

$$s_k^{(2)} = \sum_{i'} \sum_{j'} \mu_{i'j'}(\mathbf{S}, \mathbf{D}) \cdot r_{i'j'}^m \cdot p_{k|i'j'}^m \tag{67}$$

$$d_k^{(2)} = \sum_{i'} \sum_{j'} \mu_{i'j'}(\mathbf{S}, \mathbf{D}) \cdot r_{i'j'}^f \cdot p_{k|i'j'}^f \tag{68}$$

where the number of parents $\mu_{i'j'}(\mathbf{S}, \mathbf{D})$ are generated by men in class i' in the father generation, $s_{i'}$, and women in class j' in the mother generation, $d_{j'}$. These men and women in the parent generation can be estimated by men and women in the grandparent generation recursively.[16] The formulas above show a nonlinear, compound relationship between the distributions of grandparents and grandchildren. Given that there is no simple analytical form of the distribution of descendants after n generations, I simulate the two-sex long-term social reproduction effect (LSRE) in the next section and compare it with its one-sex counterpart discussed in Section 3.

In most societies, individuals tend to choose spouses with similar socioeconomic characteristics more frequently than would be expected under random mating (Schwartz 2013). Two other mating patterns, random mating and endogamous mating, which assume individuals either select mates irrespective of social background or marry only within their own social classes respectively, are less common in practice but have important theoretical implications. Formally, the random mating rule

[16] That is, $s_{i'} = \sum_i \sum_j \mu_{ij}(\mathbf{N}^m, \mathbf{N}^f) \cdot r_{ij}^m \cdot p_{i'|ij}^m$ and $d_{j'} = \sum_i \sum_j \mu_{ij}(\mathbf{N}^m, \mathbf{N}^f) \cdot r_{ij}^f \cdot p_{j'|ij}^f$. The parameters $\mathbf{N}^m$ and $\mathbf{N}^f$ refer to the number of men and women in the grandparent generation, respectively.

specifies that

$$\mu_{ij}(\mathbf{N}^m, \mathbf{N}^f) = \frac{\mathbf{N}_i^m \mathbf{N}_j^f}{(\mathbf{N}^m + \mathbf{N}^f)/2} \tag{69}$$

where $\mathbf{N}^m = \sum_i \mathbf{N}_i^m$ and $\mathbf{N}^f = \sum_j \mathbf{N}_j^f$. Compared to the assortative mating rule in equation (66), random mating assumes the number of marriages between men in class i and women in class j is only constrained by the abundance of mates.[17] For endogamous mating, we assume marriages only happen between men and women within the same social class and thus are constrained by the gender group with fewer members:

$$\mu_{ij}(\mathbf{N}^m, \mathbf{N}^f) = \begin{cases} \min(\mathbf{N}_i^m, \mathbf{N}_j^f), & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$$

To evaluate the role of assortative mating in multigenerational processes, in the next section I compare long-term multigenerational social reproduction effects estimated from various two-sex mating and mobility scenarios.

8 ILLUSTRATIVE EXAMPLES: MULTIGENERATIONAL SOCIAL MOBILITY AND REPRODUCTION IN THE UNITED STATES

8.1 Data Description

In this section, I illustrate two-generation and three-generation mobility models, with and without demography, using two sources of empirical data: (1) the IPUMS linked representative samples of

[17]Note that the random mating rule can be defined differently depending on our assumption about the constraint imposed by the size of male and female populations. Alternatively, random mating rules can be defined as

$$\mu_{ij}(\mathbf{N}^m, \mathbf{N}^f) = \frac{\mathbf{N}_i^m + \mathbf{N}_j^f}{2} \qquad \text{(arithmetic mean)}$$
$$\mu_{ij}(\mathbf{N}^m, \mathbf{N}^f) = \sqrt{\mathbf{N}_i^m \mathbf{N}_j^f} \qquad \text{(geometric mean)}$$
$$\mu_{ij}(\mathbf{N}^m, \mathbf{N}^f) = a\mathbf{N}_i^m + (1-a)\mathbf{N}_j^f, \quad 0 \le a \le 1, \qquad \text{(weighted mean)}$$
$$\mu_{ij}(\mathbf{N}^m, \mathbf{N}^f) = \mathbf{N}_i^m \qquad \text{(male dominance)}$$
$$\mu_{ij}(\mathbf{N}^m, \mathbf{N}^f) = \mathbf{N}_j^f \qquad \text{(female dominance)}$$
$$\mu_{ij}(\mathbf{N}^m, \mathbf{N}^f) = \min(\mathbf{N}_i^m, \mathbf{N}_j^f) \qquad \text{(minimum abundance)}$$

These functions are all considered as random mating because the number of marriages does not depend on parameters related to individual preferences between different class groups.

U.S. censuses (1850 to 1930) (Ruggles et al. 2019), and (2) the Panel Study of Income Dynamics (1968 to 2015) (PSID Main Interview User Manual 2019). The IPUMS linked data are constructed from linking the 1880 complete-count database to 1% samples of the 1850 to 1930 U.S. censuses of the population. The data combine samples from seven pairs of years—1850–1880, 1860–1880, 1870–1880, 1880–1900, 1880–1910, 1880–1920, and 1880–1930—in which parents' information is observed in the first census year and offspring in the second. Each year contains three independent linked samples: one of men, one of women, and one of married couples.[18] Given that the female data contain many missing cases in occupational variables, the following illustration focuses only on male mobility. Occupations in the historical census data are coded using the 1950 Census occupation classifications scheme.[19]

The empirical analysis also includes three-generation data from the Panel Study of Income Dynamics, 1968 to 2015. The PSID began in 1968 with a household sample of more than 18,000 Americans from roughly 5,000 families. Original panel members have been followed prospectively each year through 1997 and then biannually. The study follows targeted respondents according to a genealogical design. All household members recruited into the PSID in 1968 carry the PSID "gene" and are targeted for collection of detailed socioeconomic information. Members of new households created by offspring of the original targeted household heads retain the PSID "gene" themselves and become permanent PSID respondents. The PSID Family Identification Mapping System (FIMS) provides a tool to create multigenerational linked samples.[20] I supplement the analysis with simulation data to illustrate a wide range of scenarios that are theoretically important (e.g., perfect immobility or random mating) but generally not observed in empirical data.

The PSID survey asked household heads and wives to report their occupations in almost every wave of the survey. These data have been coded into detailed three-digit census categories since 1980. As part of a retrospective project, PSID created the Retrospective Occupation-Industry file by collecting three-digit occupation codes for the period 1968 to 1980 (Survey Research Center 1999).

[18]More information about the data can be found on the IPUMS website: https://usa.ipums.org/usa/linked_data_samples.shtml

[19]See the original occupation codes at the IPUMS website: https://usa.ipums.org/usa-action/variables/OCC1950#codes_section.

[20]More information abou the Family Identification Mapping System can be found on the PSID website http://simba.isr.umich.edu/FIMS/.

I merged these data with cross-year individual files. The occupational variables in the 1968 to 2001 PSID file were originally coded using Census 1970 classification codes, and those in the 2003 to 2015 file were coded using Census 2000 classification codes. Following Hauser (1980), I converted these three-digit occupations into five major occupational groups (upper nonmanual, lower nonmanual, upper manual, lower manual, and farming).[21] Because the longitudinal data provide multiple-year observations of each respondent, I use the mode of the cross-year occupational variables (i.e., the occupation that appears most often) to define a person's lifetime occupation.

The reproduction rates by fathers' social class are calculated from the average number of sons in the data. Strictly speaking, such a measure is not equivalent to the typical Gross Reproduction Rate (GRR) measure, because GRR is defined as the average number of sons who would be born to a man during his lifetime if he lives through his childbearing years and conforms to the age-specific reproduction rates of a given year.[22] The surveys omitted sons who died during young childhood before they were recorded by the next census or before they became a PSID respondent. Also, this measure may underestimate some fathers' fertility if they were not linked to some of their sons in the historical censuses or if they did not live together in the PSID households.

8.2 Empirical Results

Table 1 presents transition matrices in Markov chain mobility models for the IPUMS linked historical census data and the contemporary PSID sample. Following the tradition of mobility table research, I present father's occupation as the row variable and son's occupation as the column variable. The table also includes GRR by father's social class for all fathers in the sample as a synthetic cohort. The mobility probabilities and GRRs are estimated from the multinomial logistic

[21]For broad occupational groups based on the Census 1970 classification codes, I define upper nonmanual as professional and administrative workers (codes 1/246); lower nonmanual as sales and clerical workers (codes 260/396); upper manual as craftsmen (codes 401/696); lower manual as operatives, laborers, and service workers (codes 701/785, 901/984); and farming as farmers, farm managers, and farm laborers (codes 801/846). For broad occupational groups based on the Census 2000 codes, upper nonmanual includes managerial and professional workers (codes 1/354); lower nonmanual includes service, clerical, and sales occupations (codes 360/593); upper manual includes construction, extraction, maintenance, and production workers (codes 620/896); lower manual includes transportation and material moving workers (codes 900/975); and farming includes all the farming- and fishing-related workers (codes 600/613). The Census 1970 and 2000 occupational classifications can be found at the IPUMS website: https://usa.ipums.org/usa-action/variables/OCC#codes_section.

[22]If the sex ratio at birth is assumed at 1, GRR is approximately half of the Total Fertility Rates.

regressions and Poisson regressions, respectively (see Appendix Tables S2 and S4). All results in this section ignore the age structure of the population, as age-classified models require mobility, fertility, and mortality rates by age group and social class, but such estimates are often unreliable in surveys due to small sample sizes. For the historical sample, over 55% of fathers belong to the farming population. This number declines to 6% in the contemporary sample. For both samples, sons of upper nonmanual fathers are most likely to stay in the same occupation as their fathers: the chances of immobility are 31.1% and 38.5%, respectively. In the late 19$^{\text{th}}$ and early 20$^{\text{th}}$ centuries, more than half of sons born into lower manual families inherited their father's occupations, as compared to 30.1% of their counterparts in the contemporary sample. Reproduction rates have declined over time, from higher than 2.6 sons per father at the beginning of the historical sample to less than 1.6 sons per father in the most recent data. This trend is even more pronounced for farmers. The social class gradient in fertility has also become less remarkable over time. The gap in GRR between farmers and upper nonmanual workers decreased from 0.5 in the historical sample to 0.1 in the contemporary sample. In both datasets, sons of upper nonmanual fathers are more likely to become upper nonmanual workers than are sons of lower nonmanual fathers. Specifically, the immobility probability for sons born to upper nonmanual fathers is 0.311 and 0.385, respectively, whereas the upward mobility probability for sons born to lower nonmanual fathers is 0.236 and 0.241, respectively.

Table 2 shows similar estimates from three-generation mobility transition matrices by taking into account grandfathers' occupational class. Numbers in the mobility table refer to the percentage of sons who would achieve a certain occupational group conditional on the father's and grandfather's occupations. Numbers in the column of reproduction rates refer to the number of sons of a father conditional on his own and his father's occupations. These numbers are estimated from the multinomial logistic regressions and Poission regressions shown in Appendix Tables S3 and S5. In both samples, the highest GRR is observed in families in which both fathers and grandfathers are farmers. Some estimates may not be reliable, for example, the GRR of farmers with fathers in lower nonmanual occupations, given the small number of such families (N = 1) in the PSID sample. For illustration purposes of the methods, I ignore such possible inaccuracies, but future research should be cautious about fertility estimates from surveys.

Table 3 shows mobility effects and social reproduction effects estimated from the mobility matrices and GRRs in Tables 1 and 2. The ratio measure of mobility effects is defined as the probability of sons from upper nonmanual fathers divided by that of sons from lower nonmanual fathers to become upper nonmanual workers. Note that the result shows only one of many possible choices of the target group and the baseline group in the definitions of mobility and social reproduction effects. The measures of social reproduction effects are defined analogously, except that the outcome measure is the number of sons in upper nonmanual occupations rather than the mobility probability of sons. The net and total effects are the same for the father generation but may differ for the grandfather generation. In particular, the net effect of grandfathers is estimated by assuming the father and grandfather are in the same occupation group, whereas the total effect of grandfathers is estimated by assuming the father's occupation is uncontrolled. The net effects are all bigger than the total effects because the former compares differences in descendants from advantaged and disadvantaged families under a more extreme condition. The total mobility and social reproduction effects of grandfathers are smaller than those of fathers because of the "regression toward the mean" phenomenon. Such a trend does not exist in comparing the net effect of fathers and grandfathers.

Table 4 shows the total social reproduction effect and the effect decomposition based on difference measures. The total effect of parents (or grandparents) reflects differences in the total number of sons (or grandsons) in upper nonmanual occupations from fathers (or grandfathers) who are in upper nonmanual occupations versus lower nonmanual occupations. For example, in the historical census data, an upper nonmanual grandfather produces 0.431 more grandsons in upper nonmanual occupations than does a lower nonmanual grandfather. The Kitagawa decomposition shows the parts of the social reproduction effects associated with fertility (53.4%) and mobility (46.6%). The proportion explained by the fertility effect is small in the contemporary data (3.5%) because of the small difference in GRR between men in upper nonmanual and lower nonmanual occupations. As a result, the mobility effect accounts for over 96% of the total social reproduction effect. The Das Gupta decomposition method further partitions the total effect into the mobility and demography effect from the grandparent on the parent generation and from the grandparent on the grandchild generation net of the parent generation. For example, the total effect of grandparents contains the effect of grandparents' own fertility, the effect of grandparents on parents' fertility, the effect of grandparents on parents' mobility, and the effect of grandparents on grandchildren's mobility. In the historical data, the mobility advantage of grandchildren born to upper nonmanual grandparents relative to those born to lower nonmanual grandparents to achieve upper nonmanual occupations themselves accounts for 41.9% of the total social reproduction effect of grandparents. This proportion declines to 30.5% in the contemporary data. Most interestingly, in the historical data, most influences from grandparents to grandchildren operate through the influence of grandparents' occupation on their own fertility and the influence of grandparents' occupation on grandchildren's social mobility. In the contemporary data, however, most influences from grandparents to grandchildren work through the influence of grandparents' occupation on parents' mobility and that on grandchildren's mobility. Consistent with our ratio measures in Table 3, parent or grandparent effects of any kind are smaller in the contemporary data than in the historical data.

Table 5 shows the potential effect of one generation in upper nonmanual occupations on the distribution of descendants in succeeding generations. The long-term SRE based on a ratio measure suggests the degree to which a man in an upper nonmanual occupation, compared to one in a lower nonmanual occupation, has descendants in upper nonmanual occupations. In the historical sample, the effect begins with a value of 1.46 and eventually converges to equilibrium at 1.16. Thus, differential reproduction rates that favor upper nonmanual men further amplify the effects of intergenerational transmission of status in the historical sample. By contrast, in the contemporary data, an upper nonmanual man has approximately 1.02 times as many upper nonmanual descendants as his counterpart in lower nonmanual occupations in the long run. Upper nonmanual men produce significantly more descendants in upper nonmanual occupations in the first few generations, but this advantage almost disappears in the long run. This result can be explained by the fact that GRR for men in upper nonmanual occupations is slightly higher than that for men in lower nonmanual occupations (shown in Table 1).

Tables 6 and 7 illustrate results from two-sex models. Table 6 shows two-sex assortative mating "force of attraction," which represents the likelihood that men and women from different occupation groups will form unions, sometimes known as an indicator of preferences between two occupation groups. The number of marriages between husbands and wives in different occupations in the PSID data are included in parentheses. These numbers are observed in the first generation but are estimated in following generations as a function of the force of attraction and the constraint imposed by the size of occupation groups for the male and female populations.

Table 7 shows results of the total social reproduction of parents under the assumption of three alternative mating rules. The random mating rule assumes individuals sort into marriages irrespective of their occupational characteristics. The number of marriages is only constrained by the abundance of eligible men and women in each pair of occupations. The endogamous mating rule assumes men and women marry only within their own occupation groups. The number of marriages between men and women in different occupations is zero, and the number of marriages between men and women of the same occupation is constrained by the group with fewer members. I also consider three mobility rules: two-sex mobility assumes both parents equally influence their offspring's occupational attainment, same-sex mobility assumes individuals are influenced by their same-sex parent only, and immobility assumes sons inherit occupations from their fathers and daughters inherit occupations from their mothers. Results in Table 7 suggest the strongest parent effect emerges when men and women marry within their own occupation group and offspring inherit occupations perfectly from their parents. The ratio measure for this scenario is undefined because it is impossible for offspring born to lower nonmanual parents to become upper nonmanual workers. The effect is smallest when people mate randomly and offspring's mobility is influenced only by their same-sex parent. Overall, the two-sex results emphasize the consequences of interactions between men and women and the joint effect of fathers and mothers for determining the occupational distribution of their offspring.

9 CONCLUSIONS

This paper provides an integrated methodological framework that allows researchers to analyze the combined effects of social mobility and demography in the processes of multigenerational social inequality among families. It shifts the focus from a pure probabilistic view on individuals' mobility probabilities to a distributional view that emphasizes the number of offspring and descendants who vary in their social class in succeeding generations. Families who have more high-status offspring may be different from families whose offspring have a higher probability of achieving high status, because the joint effect of fertility and mortality may operate against families' advantages in social mobility. The moderating effects of demographic forces often accumulate over time, as the intergenerational reproduction of families is a dual endeavor to achieve both reproductive success and status inheritance across generations. I illustrated how to define and estimate various types of multigenerational processes and effects with and without the role of demography from short- and long-term perspectives. More specifically, these methodological issues include differences between two-generation and multigenerational transition matrices, net and total social reproduction effects from one generation to succeeding generations, the effect decomposition, equilibrium states of long-term multigenerational effects, three types of heterogeneity in multigenerational mobility, and two-sex versus one-sex multigenerational models and their different implications for population dynamics. Careful and creative use of these models with appropriate multigenerational data will advance our knowledge of family processes in the past and help forecast trends in the future.

Despite their advantages over conventional two-generation mobility methods based on Markov chains, the multigenerational models proposed in this paper are far from complete. I outline several promising directions for future research to consider, when more refined statistical demographic techniques are available for modeling the complex interactions among family members.

First, all models discussed in this paper assume a single, constant measure of socioeconomic status for each generation, ignoring changes in parents' status across their own and their children's life spans. From a life-course perspective, common indicators of social status—including education, employment, and earnings prospects—evolve over time. Life-cycle changes in labor supply, human capital accumulation, consumption, and nonmarket returns to education jointly shape individuals' life trajectories and their offspring's childhood skill formation (Heckman 1976; Heckman et al. 2013). Ignoring parents' and offspring' life cycles in the estimation of intergenerational association may lead to "life-cycle bias" (Mazumder 2005; Haider and Solon 2006). Furthermore, shared lifetimes, namely years during which two or more generations overlap, often vary across families and may predict the cumulative amount of influence from one generation to another (Song and Mare forthcoming). The complex linkage between within-generation status changes with age and between-generation transmission of statuses warrants future consideration, given the growth of precarious work and earnings instability in the U.S. labor market (Gottschalk et al. 1994; Kalleberg 2009).

Second, models in this paper do not address problems of causal inference, especially mechanisms and identification issues in estimating causal effects of grandparents and other kin. A fast-growing literature in sociology and economics points to causality as a central problem in multigenerational research and offers various approaches to gauging biases in traditional association measures. Anderson et al. (2018) provide a meta-analysis of recent studies on grandparent effects on education, showing a wide range of effect estimates that vary by social contexts, analytical methods, and the way the concept of grandparent effects is operationalized. Other problems, such as collider bias, unobserved confounders, and survivorship selection (e.g., Breen 2018; Sharkey and Elwert 2011; Song 2016), have also appeared in studies that aim to make causal claims of multigenerational influence using counterfactual analyses, causal graphical models, and methods based on inverse probability treatment weighting (IPTW).

Third, models in this paper ignore the complex role of migration in shaping multigenerational influences and population dynamics. Without migration, all changes in a closed population result only from births and deaths, as illustrated in the social reproduction model in equation (11). Yet, migration, both internal and international, may change the composition of a population, household structure, and relationships among extended family members in origin and destination places. Zeng and Xie (2014) provide an example of grandparent influences in rural China, where many are skip-generation households with children living with their grandparents while their parents leave to work in urban areas. More work is needed to test whether and how the mobility process is interdependent with migration, even many generations back, as well as how big events, such as mass migration and refugee resettlement, influence social mobility for descendants of migrant and native-born populations.

Fourth, all socioeconomic measures embrace some degree of uncertainty that may result from random measurement errors or systematic biases. Substantive and methodological issues in the latent structure of variables are an old topic in sociology (e.g., Coleman 1964b), but they have recently been highlighted in analyses of multigenerational mobility (Solon 2018; Torche and Corvalan 2018; Clark 2014). Random noise in socioeconomic measures may lead to attenuation bias, the classical errors-in-variables problem, which shrinks estimated intergenerational correlations toward zero. Yet, no matter how accurately measured, indicators like income, occupation, and education are always a proxy for the underlying concept of "social status." In fact, almost all methodological issues described in this paper are still valid in models that incorporate measurement errors or a latent structure of underlying variables. However, as Singer and Spilerman (1976: 454) note, dealing with the hidden structure of variables in Markov chains would lead to a considerable increase in complexity in both theory and methods.

All of these issues lie outside the scope of this paper, but they are central to the study of social stratification and mobility and to a better understanding of the social mechanisms that govern continuity and changes within families, dynasties, and populations. I leave these important modifications and challenges to future research.

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Table 1. Mobility Transition Matrix and Gross Reproduction Rates

		Historical Social Mobility						
	Gross Reproduction Rate (GRR)	Son's Occupation						
Father's Occupation		1	2	3	4	5	Total	N
1. Upper nonmanual	2.6	31.1	32.2	8.0	20.9	7.8	100.0	8,564
2. Lower nonmanual	2.4	23.6	44.9	8.8	17.7	5.0	100.0	3,575
3. Upper manual	2.6	10.0	21.6	24.1	36.4	7.9	100.0	9,609
4. Lower manual	2.6	7.7	15.8	9.8	58.6	8.1	100.0	13,306
5. Farming	3.1	7.0	2.9	2.4	9.2	78.5	100.0	43,079
N		8,514	9,806	5,656	17,684	36,473		78,133
		Contemporary Social Mobility from PSID						
	Gross Reproduction Rate (GRR)	Son's Occupation						
Father's Occupation		1	2	3	4	5	Total	N
1. Upper nonmanual	1.5	38.5	22.5	22.5	14.9	1.5	100.0	910
2. Lower nonmanual	1.4	24.1	28.8	26.2	20.3	0.6	100.0	473
3. Upper manual	1.4	16.7	21.5	38.6	22.2	1.0	100.0	1,473
4. Lower manual	1.4	12.7	22.0	33.2	30.1	2.1	100.0	1,024
5. Farming	1.6	13.4	10.7	37.8	20.2	17.9	100.0	262
N		875	911	1,337	920	99		4,142

Source: IPUMS Linked Representative Samples, 1850–1930 (final data release June 2010); Panel Study of Income Dynamics, 1968–2015.
Notes: The two-generation transition matrices show percentages converted from mobility probabilities, e.g., $p_{Y_2=j|Y_1=i}$; namely, the son of a father in social position i ends up in position j (see equation (1)).

Table 2. Three-Generation Mobility Transition Matrices

Grandfather's Occupation	Father's Occupation	Gross Reproduction Rate (GRR)	Historical Social Mobility: Son's Occupation 1	2	3	4	5	Total	N
1. Upper nonmanual	1. Upper nonmanual	2.5	37.6	34.1	7.4	16.1	4.7	100.0	3,204
	2. Lower nonmanual	2.3	28.9	46.5	8.0	13.4	3.2	100.0	934
	3. Upper manual	2.5	16.3	27.2	21.6	29.5	5.3	100.0	660
	4. Lower manual	2.5	13.8	21.6	10.9	48.0	5.7	100.0	753
	5. Farming	2.8	17.5	6.2	4.6	12.0	59.7	100.0	1,034
	N		1,869	1,934	585	1,322	875		6,585
2. Lower nonmanual	1. Upper nonmanual	2.4	33.6	37.0	7.9	17.6	3.9	100.0	403
	2. Lower nonmanual	2.2	25.3	49.5	8.4	14.3	2.6	100.0	637
	3. Upper manual	2.4	14.0	28.4	22.3	31.1	4.2	100.0	156
	4. Lower manual	2.3	11.8	22.4	11.2	50.1	4.5	100.0	228
	5. Farming	2.6	17.4	7.6	5.5	14.6	54.9	100.0	150
	N		371	571	154	347	131		1,574
3. Upper manual	1. Upper nonmanual	2.5	25.8	32.3	12.2	22.7	7.0	100.0	1,135
	2. Lower nonmanual	2.3	19.7	43.8	13.1	18.7	4.7	100.0	451
	3. Upper manual	2.5	9.1	21.1	29.2	34.1	6.5	100.0	4,648
	4. Lower manual	2.5	7.6	16.5	14.5	54.5	6.8	100.0	2,051
	5. Farming	2.8	9.2	4.5	5.8	12.9	67.7	100.0	1,573
	N		1,107	1,954	1,944	3,248	1,605		9,858
4. Lower manual	1. Upper nonmanual	2.6	23.6	32.2	8.6	29.1	6.5	100.0	1,142
	2. Lower nonmanual	2.4	18.1	44.0	9.4	24.1	4.4	100.0	591
	3. Upper manual	2.6	8.4	21.1	20.7	43.8	6.0	100.0	1,797
	4. Lower manual	2.5	6.4	15.0	9.3	63.6	5.7	100.0	6,056
	5. Farming	2.8	8.7	4.7	4.3	17.2	65.1	100.0	2,596
	N		1,139	2,038	1,202	5,557	2,246		12,182
5. Farming	1. Upper nonmanual	2.8	28.2	29.3	6.7	22.9	12.9	100.0	2,680
	2. Lower nonmanual	2.6	22.5	41.4	7.5	19.7	9.0	100.0	962
	3. Upper manual	2.8	10.9	20.9	17.4	37.7	13.0	100.0	2,348
	4. Lower manual	2.8	8.5	15.1	8.0	55.7	12.7	100.0	4,218
	5. Farming	3.1	6.5	2.6	2.0	8.4	80.4	100.0	37,726
	N		4,028	3,309	1,771	7,210	31,616		47,934

Grandfather's Occupation	Father's Occupation	Gross Reproduction Rate (GRR)	Contemporary Social Mobility from PSID: Son's Occupation 1	2	3	4	5	Total	N
1. Upper nonmanual	1. Upper nonmanual	1.4	48.1	22.5	17.7	10.6	1.1	100.0	234
	2. Lower nonmanual	1.4	33.4	30.3	21.2	14.6	0.4	100.0	63
	3. Upper manual	1.4	26.4	25.3	31.2	16.4	0.7	100.0	109
	4. Lower manual	1.4	21.1	27.4	27.7	22.4	1.5	100.0	63
	5. Farming	1.5	25.6	17.7	29.4	15.2	12.1	100.0	13
	N		179	119	110	68	6		482
2. Lower nonmanual	1. Upper nonmanual	1.3	45.2	25.1	14.9	13.5	1.3	100.0	131
	2. Lower nonmanual	1.3	30.7	33.0	17.6	18.2	0.5	100.0	69
	3. Upper manual	1.3	24.6	27.9	26.1	20.7	0.8	100.0	89

	4. Lower manual	1.3	19.0	29.3	22.5	27.4	1.7	100.0	66
	5. Farming	1.4	23.5	19.2	24.2	18.9	14.1	100.0	1
	N		115	100	70	67	4		356
3. Upper manual	1. Upper nonmanual	1.5	34.2	25.0	25.0	14.0	1.8	100.0	206
	2. Lower nonmanual	1.4	22.1	31.2	27.9	18.0	0.7	100.0	125
	3. Upper manual	1.4	16.5	24.7	38.7	19.0	1.0	100.0	422
	4. Lower manual	1.4	12.9	26.0	33.5	25.4	2.2	100.0	200
	5. Farming	1.5	15.2	16.3	34.5	16.7	17.4	100.0	9
	N		195	248	320	184	15		962
4. Lower manual	1. Upper nonmanual	1.4	32.5	23.6	23.6	18.9	1.4	100.0	196
	2. Lower nonmanual	1.4	20.7	29.0	26.0	23.8	0.6	100.0	141
	3. Upper manual	1.4	15.4	22.8	35.9	25.1	0.8	100.0	466
	4. Lower manual	1.4	11.7	23.5	30.3	32.7	1.7	100.0	390
	5. Farming	1.5	14.6	15.6	33.1	22.8	13.9	100.0	44
	N		217	292	383	325	20		1,237
5. Farming	1. Upper nonmanual	1.6	30.9	15.2	32.4	19.2	2.3	100.0	143
	2. Lower nonmanual	1.6	19.8	18.9	36.0	24.4	0.9	100.0	75
	3. Upper manual	1.5	13.9	14.0	46.8	24.2	1.2	100.0	387
	4. Lower manual	1.6	10.7	14.6	40.1	32.0	2.6	100.0	305
	5. Farming	1.7	12.1	8.8	39.6	20.2	19.3	100.0	195
	N		169	152	454	276	54		1,105

Source: IPUMS Linked Representative Samples, 1850–1930 (final data release June 2010); Panel Study of Income Dynamics, 1968–2015.

Notes: The three-generation transition matrix shows percentages converted from mobility probabilities, e.g., $p_{Y_3=j|Y_2=i,Y_1=k}$; namely, the son of a father in social position i and a grandfather in social position k ends up in position j.

Table 3. Ratio Measures of Mobility Effects and Social Reproduction Effects by Comparing Upper Nonmanual and Lower Nonmanual Families in Producing Offspring in Upper Nonmanual Occupations

	Mobility Effect		Social Reproduction Effect	
	Net Effect (assuming fathers and grandfathers in the same occupation)	**Total Effect** (unconditional on fathers' occupations)	**Net Effect** (assuming fathers and grandfathers in the same occupation)	**Total Effect** (unconditional on fathers' occupations)
Historial data				
Parents	1.317	1.317	1.456	1.456
	(0.044)	(0.044)	(0.055)	(0.055)
Grandparents	1.490	1.133	1.720	1.344
	(0.085)	(0.058)	(0.109)	(0.080)
Contemporary data				
Parents	1.596	1.596	1.622	1.622
	(0.146)	(0.146)	(0.164)	(0.164)
Grandparents	1.566	1.178	1.691	1.277
	(0.195)	(0.121)	(0.233)	(0.157)

Source: IPUMS Linked Representative Samples, 1850–1930 (final data release June 2010); Panel Study of Income Dynamics, 1968–2015.
Notes: Standard errors of the predicted net and total mobility effect and social reproduction effect are estimated from 1,000 bootstrap samples. The net mobility effect refers to the ratio between the probability of achieving upper nonmanual occupations by having upper nonmanual parents rather than lower nonmanual parents (or upper nonmanual grandparents and parents versus lower nonmanual grandparents and parents). The total mobility effect is calculated from the ratio between the the probability of achieving upper nonmanual occupations by having upper nonmanual grandparents rather than lower nonmanual grandparents. For the net social reproduction effect, we compare parents (and grandparents) in upper nonmanual occupations with those in lower nonmanual occupations in producing upper nonmanual offspring (or grandchildren). For the total effect of grandparents, we compare grandparents who are in upper nonmanual occupations with those in lower nonmanual occupations in producing upper nonmanual grandchildren. The mobility effects and social reproduction effects are defined in equations (19)–(24).

Table 4. Effect Decomposition Based on Difference Measures of Social Reproduction Effects by Comparing Upper Nonmanual and Lower Nonmanual Families in Producing Offspring in Upper Nonmanual Occupations

		Kitagawa Decomposition		**Das Gupta Decomposition**			
	Total Social Reproduction Effect (%)	**Total Demography Effect (%)**	**Total Mobility Effect (%)**	**Demography Effect(1) (%)**	**Demography Effect(2) (%)**	**Mobility Effect(1) (%)**	**Mobility Effect(2) (%)**
Historical data							
Parents	0.253	0.068	0.185	0.068	-	0.185	-
	(100.0)	(26.7)	(73.3)	(26.7)		(73.3)	
Grandparents	0.431	0.230	0.201	0.146	0.085	0.020	0.181
	(100.0)	(53.4)	(46.6)	(33.8)	(19.7)	(4.6)	(41.9)
Contemporary data							
Parents	0.215	0.007	0.207	0.007	-	0.207	-
	(100.0)	(3.5)	(96.5)	(3.5)		(96.5)	
Grandparents	0.157	0.048	0.109	0.011	0.038	0.061	0.048
	(100.0)	(30.8)	(69.2)	(6.7)	(24.0)	(38.7)	(30.5)

Source: IPUMS Linked Representative Samples, 1850–1930 (final data release June 2010); Panel Study of Income Dynamics, 1968–2015.
Note: Numbers in the parentheses are percentages of the total effect explained by each of the demographic and social mobility components. The decomposition methods are described in equations (28), (35) and (37)–(40).

Table 5. Long-term Social Reproduction Effects

	Occupation in the founding generation	**Distribution of Descendants** 1. Upper nonmanual	2. Lower nonmanual	3. Upper manual	4. Lower manual	5. Farming	Long-term SRE in producing upper nonmanual descendants
Historical data							
After n generations							
1	Upper nonmanual	0.81	0.84	0.21	0.54	0.20	1.46
	Lower nonmanual	0.55	1.06	0.21	0.42	0.12	
5	Upper nonmanual	17.24	25.45	10.36	35.51	30.55	1.12
	Lower nonmanual	15.36	22.87	9.23	31.52	25.29	
10	Upper nonmanual	2,190.05	3,036.50	1,288.62	4,507.76	5,566.03	1.15
	Lower nonmanual	1,906.90	2,652.90	1,124.17	3,930.59	4,775.19	
∞	Upper nonmanual	-	-	-	-	-	1.16
	Lower nonmanual	-	-	-	-	-	
Contemporary data							
After n generations							
1	Upper nonmanual	0.56	0.33	0.33	0.22	0.02	1.62
	Lower nonmanual	0.34	0.41	0.38	0.29	0.01	
5	Upper nonmanual	1.38	1.44	1.90	1.34	0.10	1.02
	Lower nonmanual	1.35	1.40	1.85	1.31	0.09	
10	Upper nonmanual	8.31	8.63	11.41	8.05	0.57	1.02
	Lower nonmanual	8.12	8.44	11.16	7.87	0.56	
∞	Upper nonmanual	-	-	-	-	-	1.02
	Lower nonmanual	-	-	-	-	-	

Source: IPUMS Linked Representative Samples, 1850–1930 (final data release June 2010); Panel Study of Income Dynamics, 1968–2015.
Notes: Intergenerational mobility is assumed to follow a Markovian process. Similar results are valid if mobility follows higher-order Markovian processes. The long-term effect is defined as the ratio of upper nonmanual progeny per upper nonmanual ancestor over upper nonmanual progeny per lower nonmanual ancestor. The ratio = 1 means no long-term effect. The effect is defined in equation (54).

Table 6. Two-Sex Assortative Mating and Force of Attraction (age 25-60)

	Occupation, Women					
Occupation, Men	1. Upper nonmanual	2. Lower nonmanual	3. Upper manual	4. Lower manual	5. Farming	N
1. Upper nonmanual	0.858	0.723	0.181	0.253	0.018	
	(348)	(431)	(78)	(146)	(2)	1,005
2. Lower nonmanual	0.321	0.643	0.219	0.323	0.010	
	(98)	(259)	(70)	(127)	(1)	555
3. Upper manual	0.274	0.677	0.642	0.673	0.164	
	(126)	(487)	(316)	(465)	(19)	1,413
4. Lower manual	0.220	0.400	0.560	0.849	0.307	
	(95)	(263)	(259)	(537)	(35)	1,189
5. Farming	0.078	0.142	0.194	0.409	0.863	
	(13)	(27)	(33)	(77)	(69)	219
N	680	1,467	756	1,352	126	4,381

Source: Panel Study of Income Dynamics, 1968–2015.
Notes: Numbers in parentheses refer to the number of marriages within each assortative mating category. The parameter for the "force of attraction" (α_{ij}) represents the likelihood that men and women from two occupation groups will form unions. This value is a function of preferences between two occupation groups and constraints imposed by the sizes of the two groups. The force of attraction is defined in equation (66).

Table 7. Ratio Measures of Social Reproduction Effects under Different Mating and Mobility Rules

		Total Social Reproduction Effects of Upper Nonmanual vs. Lower Nonmanual Parents	
Mating Rule	**Intergenerational Mobility Rule**	**Difference measure**	**Ratio measure**
Random mating	Same-sex (father-son; mother-daughter)	0.034	2.014
	Two-sex	0.041	1.987
	Immobility (perfect inheritance)	0.160	∞
Endogamous mating	Same-sex (father-son; mother-daughter)	0.193	2.940
	Two-sex	0.234	2.900
	Immobility	0.695	∞
Assortative mating	Same-sex (father-son; mother-daughter)	0.103	3.225
	Two-sex	0.125	3.180
	Immobility (perfect inheritance)	0.356	∞

Source: Panel Study of Income Dynamics (1968–2015) and simulation data.
Notes: For the total effect of parents, we compare parents who are in upper nonmanual occupations with those in lower nonmanual occupations in producing upper nonmanual offspring. The ratio = 1 means no effect. The effect takes into account probabilities that men and women in upper nonmanual (or lower nonmanual) occupations will form unions, produce offspring, and transmit their social status to their offspring. The random mating rule assumes mating between individuals where the choice of partner is not influenced by occupations. The endogamous mating rule assumes men and women marry only within their own occupation groups. The assortative mating rule assumes individuals with similar occupations mate with one another more frequently than would be expected under a random mating rule. The same-sex mobility rule assumes individuals are influenced by their same-sex parent only (namely, sons by fathers and daughters by mothers). The two-sex mobility rule assumes individuals' occupations are influenced by occupations of both parents. The immobility rule assumes sons inherit occupations from their fathers and daughters inherit occupations from their mothers.

ONLINE APPENDIX, NOT FOR PUBLICATION

Appendix Table S1. A Summary of Intergenerational Social Mobility Research

Models	**Composition**	**Mobility**	**Demography**	**Methods**	**Exemplary Prior Research**
Classic Mobility Models	One-Sex	Two-Generation	No	Markov chain models	Prais (1955)
				Mobility tables and path analysis	Blau and Duncan (1967); Featherman and Hauser (1978)
				Loglinear models	Erikson and Goldthorpe (1992); Grusky and Hauser (1984); Hout (1988); Jonsson et al. (2009); Torche (2011); Yamaguhi (1987); Xie (1992)
				Log-log regression	Solon (1992)
	One-Sex	Multiple-Generation	No	Loglinear models	Chan and Boliver (2013)
				Survival analysis	Zeng and Xie (2014)
				Rank-rank regression	Pfeffer and Killewald (2017)
				Log-log regression	Solon (2014, 2018)
	Two-Sex	Two-Generation	No	Loglinear models	Beller (2009)
				Rank-rank regression	Chetty et al. (2014)
				Log-log regression	Lee and Solon (2009)
	Two-Sex	Multiple-Generation	No	Path analysis	Warren and Hauser (1997)
Joint Demography-Mobility Models	One-Sex	Two-Generation	Yes	Markov chain models with demography	Matras (1961); Preston (1974); Mare (1997); Mare and Maralani (2006); Maralani (2013)
	One-Sex	Multiple-Generation	Yes	Markov chain models with demography	Mare and Song (2014)
	Two-Sex	Two-Generation	Yes	Markov chain models with demography	Preston and Campbell (1997)
	Two-Sex	Multiple-Generation	Yes	Markov chain models with demography	Song and Mare (2016)

Appendix Table S2. Two-Generation Reproduction and Social Mobility Models, Historical Data

	Gross Reproduction Rate (Poisson Regression)		Mobility Model: Son's Occupation (Multinomial Logistic Regression, Base = 1. Upper nonmanual)							
			2. Lower nonmanual		3. Upper manual		4. Lower manual		5. Farming	
Father's Occupation										
2. Lower nonmanual	-0.100***	(0.02)	0.607***	(0.05)	0.371***	(0.079)	0.108*	(0.061)	-0.161*	(0.093)
3. Upper manual	0.004	(0.015)	0.732***	(0.048)	2.234***	(0.057)	1.686***	(0.048)	1.145***	(0.065)
4. Lower manual	-0.004	(0.014)	0.675***	(0.047)	1.595***	(0.06)	2.418***	(0.045)	1.432***	(0.061)
5. Farming	0.161***	(0.012)	-0.908***	(0.043)	0.280***	(0.056)	0.667***	(0.039)	3.803***	(0.047)
Intercept	0.956***	(0.011)	0.037	(0.027)	-1.355***	(0.043)	-0.394***	(0.031)	-1.388***	(0.043)
n	27,734		78,133							
Log likelihood	-57,136									
AIC	114,283		163,900							

Source: IPUMS Linked Representative Samples, 1850–1930 (final data release June 2010).
Notes: Standard errors are in parentheses. The Gross Reproduction Rates and mobility probabilities estimated from these models are presented in Table 1.
$^{*}p < .05$; $^{**}p < .01$; $^{***}p < .001$ (two-tailed test).

Appendix Table S3. Three-Generation Reproduction and Social Mobility Models, Historical Data

	Gross Reproduction Rate (Poisson Regression)		Mobility Model: Son's Occupation (Multinomial Logistic Regression, Base = 1. Upper nonmanual)							
			2. Lower nonmanual		3. Upper manual		4. Lower manual		5. Farming	
Father's Occupation										
1. Upper nonmanual (reference)										
2. Lower nonmanual	-0.058**	(0.028)	0.196**	(0.076)	0.182*	(0.11)	0.203**	(0.088)	-0.079	(0.128)
3. Upper manual	0.013	(0.017)	0.323***	(0.054)	0.878***	(0.066)	0.721***	(0.055)	0.775***	(0.068)
4. Lower manual	0.029*	(0.016)	0.411***	(0.053)	0.620***	(0.068)	1.057***	(0.053)	0.785***	(0.065)
5. Farming	0.111***	(0.015)	0.136***	(0.045)	0.183***	(0.061)	0.640***	(0.047)	1.294***	(0.054)
Grandfather's Occupation										
1. Upper nonmanual (reference)										
2. Lower nonmanual	-0.085***	(0.02)	0.574***	(0.052)	0.342***	(0.08)	0.077	(0.062)	-0.131	(0.094)
3. Upper manual	0.004	(0.016)	0.612***	(0.051)	1.907***	(0.061)	1.443***	(0.051)	0.959***	(0.068)
4. Lower manual	-0.016	(0.015)	0.545***	(0.05)	1.388***	(0.063)	2.092***	(0.048)	1.192***	(0.064)
5. Farming	0.098***	(0.013)	-0.934***	(0.048)	0.290***	(0.061)	0.469***	(0.043)	3.306***	(0.05)
Intercept	0.916***	(0.014)	-0.099***	(0.036)	-1.625***	(0.055)	-0.848***	(0.042)	-2.080***	(0.058)
n	27,734		78,133							
Log likelihood	-57,104									
AIC	114,226.5		162,099.0							

Source: IPUMS Linked Representative Samples, 1850–1930 (final data release June 2010).
Notes: Standard errors are in parentheses. The Gross Reproduction Rates and mobility probabilities estimated from these models are presented in Table 2.
$^{*}p < .05$; $^{**}p < .01$; $^{***}p < .001$ (two-tailed test).

Appendix Table S4. Two-Generation Reproduction and Social Mobility Models, Contemporary Data

	Gross Reproduction Rate (Poisson Regression)		Mobility Model: Son's Occupation (Multinomial Logistic Regression, Base = 1. Upper nonmanual)							
			2. Lower nonmanual		3. Upper manual		4. Lower manual		5. Farming	
Father's Occupation										
1. Upper nonmanual (reference)										
2. Lower nonmanual	-0.016	(0.059)	0.711***	(0.154)	0.619***	(0.157)	0.773***	(0.171)	-0.419	(0.645)
3. Upper manual	-0.032	(0.043)	0.789***	(0.122)	1.373***	(0.116)	1.230***	(0.132)	0.353	(0.387)
4. Lower manual	-0.021	(0.047)	1.083***	(0.141)	1.496***	(0.136)	1.808***	(0.145)	1.396***	(0.36)
5. Farming	0.11	(0.072)	0.312	(0.268)	1.575***	(0.215)	1.360***	(0.24)	3.514***	(0.352)
Intercept	0.375***	(0.034)	-0.535***	(0.088)	-0.535***	(0.088)	-0.945***	(0.101)	-3.219***	(0.273)
n	2,689		4,142							
Log likelihood	-3,457									
AIC	6,924.4		11,609.8							

Source: Panel Study of Income Dynamics, 1968–2015.
Notes: Standard errors are in parentheses. The Gross Reproduction Rates and mobility probabilities estimated from these models are presented in Table 1.
$^{*}p < .05$; $^{**}p < .01$; $^{***}p < .001$ (two-tailed test).

Appendix Table S5. Three-Generation Reproduction and Social Mobility Models, Contemporary Data

	Gross Reproduction Rate (Poisson Regression)		Mobility Model: Son's Occupation (Multinomial Logistic Regression, Base = 1. Upper nonmanual)							
			2. Lower nonmanual		3. Upper manual		4. Lower manual		5. Farming	
Father's Occupation										
1. Upper nonmanual (reference)										
2. Lower nonmanual	-0.059	(0.072)	0.17	(0.184)	-0.105	(0.198)	0.306	(0.214)	0.241	(0.664)
3. Upper manual	0.029	(0.058)	0.442***	(0.156)	0.686***	(0.157)	0.620***	(0.181)	0.890*	(0.507)
4. Lower manual	0.01	(0.057)	0.435***	(0.154)	0.679***	(0.154)	0.966***	(0.174)	0.703	(0.493)
5. Farming	0.129**	(0.058)	0.052	(0.171)	1.049***	(0.159)	1.033***	(0.181)	1.217***	(0.472)
Grandfather's Occupation										
1. Upper nonmanual (reference)										
2. Lower nonmanual	-0.018	(0.059)	0.659***	(0.156)	0.547***	(0.159)	0.681***	(0.174)	-0.498	(0.647)
3. Upper manual	-0.051	(0.044)	0.715***	(0.126)	1.166***	(0.12)	1.031***	(0.136)	0.142	(0.394)
4. Lower manual	-0.045	(0.049)	1.019***	(0.145)	1.272***	(0.14)	1.569***	(0.15)	1.177***	(0.369)
5. Farming	0.035	(0.076)	0.387	(0.278)	1.136***	(0.226)	0.985***	(0.251)	3.065***	(0.387)
Intercept	0.354***	(0.051)	-0.757***	(0.131)	-1.000***	(0.136)	-1.509***	(0.16)	-3.818***	(0.458)
n	2,690									
Log likelihood	-3,452.0									
AIC	6,921.9									

Source: Panel Study of Income Dynamics, 1968–2015.
Notes: Standard errors are in parentheses. The Gross Reproduction Rates and mobility probabilities estimated from these models are presented in Table 2.
$^{*}p < .05$; $^{**}p < .01$; $^{***}p < .001$ (two-tailed test).

APPENDIX: R Codes Used in the Analysis of PSID Data

```
library(readstata13)
library(tidyr)
library(dplyr)
library(expm)
library(nnet)
library(reshape)
require(boot)

psid.male <- read.dta13("psid_mobility.dta", nonint.factors=T) %>%
  select(c(f_id, sex, occ, occ_f, occ_m, occ_ff, occ_fm, occ_mf, occ_mm, occ_gf, sex)) %>% drop_na(occ, occ_f, occ_gf) %>% filter(sex==1)

# Table 1
# Describe 2-generation mobility table (transition matrix)

summary(m1 <- multinom(occ ~ relevel(as.factor(occ_f), ref = "1"), data = psid.male))

data.2g <- cbind(psid.male, fitted=fitted(m1))
mobility2g <- data.2g %>%
  group_by(occ_f) %>%
  summarise(son1=mean(fitted.1), son2=mean(fitted.2), son3=mean(fitted.3), son4=mean(fitted.4), son5=mean(fitted.5))
with(psid.male, addmargins(table(occ_f, occ)))

# Describe fertility by occupation

sons.count <- psid.male %>% filter(f_id != 0) %>% arrange(-f_id) %>% group_by(f_id, occ_f) %>% summarise(sons.count=n())

summary(m2 <- glm(sons.count ~ relevel(as.factor(occ_f), ref = "1"), family="poisson", data=sons.count))
GRR1 <- exp(c(0, rep(coefficients(m2)[1],4))+coefficients(m2))

# Table 2
# Describe 3-generation mobility table

summary(m3 <- multinom(occ ~ relevel(as.factor(occ_gf), ref = "1")+relevel(as.factor(occ_f), ref = "1"), data = psid.male))
data.3g <- cbind(psid.male, fitted=fitted(m3))
mobility3g <- data.3g %>%
  group_by(occ_gf, occ_f) %>%
  summarise(son1=mean(fitted.1), son2=mean(fitted.2), son3=mean(fitted.3), son4=mean(fitted.4), son5=mean(fitted.5))
with(psid.male, addmargins(table(occ_f, occ, occ_gf)))

# Describe fertility by occupation
sons.count2 <- psid.male %>% filter(f_id != 0) %>% arrange(-f_id) %>% group_by(f_id, occ_gf, occ_f) %>% summarise(sons.count2=n())

summary(m4 <- glm(sons.count2 ~ relevel(as.factor(occ_gf), ref = "1")+relevel(as.factor(occ_f), ref = "1"), family="poisson", data=sons.count2))
intercept <- coefficients(m4)[1]
gf_coef <- c(0, coefficients(m4)[2:5])
f_coef <- c(0, coefficients(m4)[6:9])

GRR2 <- exp(intercept) * (exp(gf_coef) %x% exp(f_coef))

# Table 3 Mobility effect, net SRE, Total SRE

# net and total mobility effect of p
mobility2g <- as.matrix(mobility2g[1:5, 2:6])
mobility2g[1,1]/mobility2g[2,1]

```

```
# net mobility effect of gp
mobility3g <- as.matrix(mobility3g[1:25, 3:7])
mobility3g[1,1]/mobility3g[7,1] #assume p and gp in the same class

# total mobility effect of gp
G0.1 <- c(1,0,0,0,0)
G0.2 <- c(0,1,0,0,0)
(G0.1 %*% mobility2g %*% mobility3g[1:5,])[1,1]/(G0.2%*%mobility2g%*%mobility3g[6:10,])[1,1]

# SRE of parents
SRE.f <- (GRR1[1]*mobility2g[1,1])/(GRR1[2]*mobility2g[2,1])

# NSRE of grandparents
NSRE.gf <- (GRR2[1]*mobility3g[1,1])/(GRR2[7]*mobility3g[7,1])

# TSRE of grandparents

G1.1 <- G0.1 %*% diag(GRR1) %*% mobility2g
G2.1 <- G1.1 %*% diag(GRR2[1:5]) %*% mobility3g[1:5,]

G1.2 <- G0.2 %*% diag(GRR1) %*% mobility2g
G2.2 <- G1.2 %*% diag(GRR2[6:10]) %*% mobility3g[6:10,]

TSRE.gf <- G2.1[1]/G2.2[1]

# bootstrap standard errors

bs <- function(formula1, formula2, formula3, formula4, data, indices) {
  d1 = data[indices,]

  m1 = multinom(formula1, data=d1, maxit=1000, trace=FALSE)
  data.2g <- cbind(d1, fitted=fitted(m1))
  mobility2g <- data.2g %>% group_by(occ_f) %>%
    summarise(son1=mean(fitted.1), son2=mean(fitted.2), son3=mean(fitted.3), son4=mean(fitted.4), son5=mean(fitted.5))

  sons.count <- d1 %>% filter(f_id != 0) %>% arrange(-f_id) %>%
    group_by(f_id, occ_f) %>% summarise(sons.count=n())
  m2 = glm(formula2, family="poisson", data=sons.count, maxit=1000, trace=FALSE)
  GRR1 <- exp(c(0, rep(coefficients(m2)[1],4))+coefficients(m2))

  m3 = multinom(formula3, data=d1, maxit=1000, trace=FALSE)
  data.3g <- cbind(d1, fitted=fitted(m3))
  mobility3g <- data.3g %>% group_by(occ_gf, occ_f) %>%
    summarise(son1=mean(fitted.1), son2=mean(fitted.2), son3=mean(fitted.3), son4=mean(fitted.4), son5=mean(fitted.5))

  sons.count2 <- d1 %>% filter(f_id != 0) %>% arrange(-f_id) %>% group_by(f_id, occ_gf, occ_f) %>% summarise(sons.count2=n())
  m4 = glm(formula4, family="poisson", data=sons.count2, maxit=1000, trace=FALSE)
  GRR2 <- exp(coefficients(m4)[1]) * (exp(c(0, coefficients(m4)[2:5])) %x% exp(c(0, coefficients(m4)[6:9])))

  mobility2g = as.matrix(mobility2g[1:5, 2:6])
  mobility.f = mobility2g[1,1]/mobility2g[2,1]

  mobility3g = as.matrix(mobility3g[1:25, 3:7])
  n.mobility.gf = mobility3g[1,1]/mobility3g[7,1]

  G0.1 <- c(1,0,0,0,0); G0.2 <- c(0,1,0,0,0)
  t.mobility.gf = (G0.1 %*% mobility2g %*% mobility3g[1:5,])[1,1]/(G0.2%*%mobility2g%*%mobility3g[6:10,])[1,1]

  SRE.f = (GRR1[1]*mobility2g[1,1])/(GRR1[2]*mobility2g[2,1])
  NSRE.gf = (GRR2[1]*mobility3g[1,1])/(GRR2[7]*mobility3g[7,1])

```

```
  G1.1 <- G0.1 %*% diag(GRR1) %*% mobility2g
  G2.1 <- G1.1 %*% diag(GRR2[1:5]) %*% mobility3g[1:5,]
  G1.2 <- G0.2 %*% diag(GRR1) %*% mobility2g
  G2.2 <- G1.2 %*% diag(GRR2[6:10]) %*% mobility3g[6:10,]
  TSRE.gf = G2.1[1]/G2.2[1]

  estimates = rbind(mobility.f, SRE.f, n.mobility.gf, t.mobility.gf, NSRE.gf, TSRE.gf)

  return(t(estimates))
}

# enable parallel

cl <- makeCluster(2)
clusterExport(cl, "multinom")

# 1000 replications
set.seed(1984)

#system.time(boot(data=psid.male, statistic=bs, R=1000, parallel = "multicore", ncpus=2,
    formula=occ ~ relevel(as.factor(occ_f), ref = "1")))

results <- boot(
  data=ipums, statistic=bs, R=1000, parallel = "multicore", ncpus=2, cl=cl, formula1=occ ~
      relevel(as.factor(occ_f), ref = "1"), formula2=sons.count ~ relevel(as.factor(occ_f),
      ref = "1"), formula3=occ ~ relevel(as.factor(occ_gf), ref = "1")+relevel(as.factor(occ
      _f), ref = "1"),
  formula4=sons.count2 ~ relevel(as.factor(occ_gf), ref = "1")+relevel(as.factor(occ_f), ref
       = "1")
)


# Table 4
# Kitagawa SRE decomposition of SRE.f
kita.demo.eff.f <- (GRR1[1]-GRR1[2])*(mobility2g[1,1]+mobility2g[2,1])/2
kita.mobi.eff.f <- (GRR1[1]+GRR1[2])/2*(mobility2g[1,1]-mobility2g[2,1])

# Kitagawa SRE decomposition of TSRE.gf
kita.demo.eff.gf <- sum((GRR1[1]*GRR2[1:5]-GRR1[2]*GRR2[5+1:5])*(mobility2g[1,1:5]*
    mobility3g[1:5,1]+mobility2g[2,1:5]*mobility3g[5+1:5,1])/2)
kita.mobi.eff.gf <- sum((GRR1[1]*GRR2[1:5]+GRR1[2]*GRR2[5+1:5])/2*(mobility2g[1,1:5]*
    mobility3g[1:5,1]-mobility2g[2,1:5]*mobility3g[5+1:5,1]))

# Das Gupta SRE decomposition of TSRE.gf
r1 <-GRR1[1]; r1prime <- GRR1[2]
r2 <- GRR2[1:5]; r2prime <- GRR2[5+1:5]
p1 <- mobility2g[1,1:5]; p1prime <- mobility2g[2,1:5]
p2 <- mobility3g[1:5,1]; p2prime <- mobility3g[5+1:5,1]

das.demo.eff.1.gf <-
  sum(((p1*r2*p2+p1prime*r2prime*p2prime)/4
      +(p1*r2*p2prime+p1*r2prime*p2+p1prime*r2*p2+p1prime*r2prime*p2+p1prime*r2*p2prime+p1*
          r2prime*p2prime)/12)*(r1-r1prime))
das.demo.eff.2.gf <-
  sum(((p1*r1*p2+p1prime*r1prime*p2prime)/4
      +(p1*r1*p2prime+p1*r1prime*p2+p1prime*r1*p2+p1prime*r1prime*p2+p1prime*r1*p2prime+p1*
          r1prime*p2prime)/12)*(r2-r2prime))
das.mobi.eff.1.gf <-
  sum(((r1*r2*p2+r1prime*r2prime*p2prime)/4
      +(r1*r2*p2prime+r1*r2prime*p2+r1prime*r2*p2+r1prime*r2prime*p2+r1prime*r2*p2prime+r1*
          r2prime*p2prime)/12)*(p1-p1prime))
das.mobi.eff.2.gf <-
  sum(((r1*r2*p1+r1prime*r2prime*p1prime)/4
      +(r1*r2*p1prime+r1*r2prime*p1+r1prime*r2*p1+r1prime*r2prime*p1+r1prime*r2*p1prime+r1*
          r2prime*p1prime)/12)*(p2-p2prime))

```

```r
# Table 5 Long-term SRE (we assume mobility is Markovian)

C <- diag(GRR1) %*% mobility2g

G1.1 <- G0.1 %*% C
G2.1 <- G1.1 %*% C
G5.1 <- G0.1 %*% (C %^% (5))
G10.1 <- G0.1 %*% (C %^% (10))

G1.2 <- G0.2 %*% C
G2.2 <- G1.2 %*% C
G5.2 <- G0.2 %*% (C %^% (5))
G10.2 <- G0.2 %*% (C %^% (10))

eL <- eigen(t(C))  #left eigenvector
L <- eL$values
V <- eL$vectors
G1.1 %*% V %*% solve(t(V)%*%V)
G1.2 %*% V %*% solve(t(V)%*%V)

# Table 6 Two-sex force of attraction, #marriages
psid <- read.dta13("psid_mobility.dta", nonint.factors=T) %>% select(c(f_id, m_id, occ, occ_f, occ_m, sex)) %>% drop_na(occ, occ_f, occ_m)

child.count <- psid %>% filter(f_id != 0 | m_id != 0) %>% arrange(-f_id, -m_id) %>% group_by(f_id, m_id, occ_f, occ_m) %>% summarise(child.count=n())
summary(m5 <- glm(child.count ~ relevel(as.factor(occ_f), ref = "1")+relevel(as.factor(occ_m), ref = "1"), family="poisson", data=child.count))
intercept <- coefficients(m5)[1]
f_coef <- c(0, coefficients(m5)[2:5])
m_coef <- c(0, coefficients(m5)[6:9])

GRR.son <- GRR.daughter <- exp(intercept) * (exp(f_coef) %x% exp(m_coef))

mobility.samesex.son <- with(filter(psid, sex==1), prop.table(table(occ_f, occ), 1))
mobility.samesex.daughter <- with(filter(psid, sex==2), prop.table(table(occ_m, occ), 1))

mobility.samesex.son <- matrix(rep(mobility.samesex.son,each=5), ncol=5)
mobility.samesex.daughter <- matrix(rep(t(mobility.samesex.daughter),5) , ncol=5, byrow=TRUE)

mobility.2sex.son <- with(filter(psid, sex==1), ftable(prop.table(table(occ_f, occ_m, occ), c(1,2))))
mobility.2sex.daughter <- with(filter(psid, sex==2), ftable(prop.table(table(occ_f, occ_m, occ), c(1,2))))

mobility.perfect <- diag(rep(1, 5))
mobility.perfect.son <- matrix(rep(mobility.perfect,each=5), ncol=5)
mobility.perfect.daughter <- matrix(rep(t(mobility.perfect),5) , ncol=5, byrow=TRUE)

N.male.0 <- apply(with(psid, table(occ_f, occ_m)), 1, sum)
N.female.0 <- apply(with(psid, table(occ_f, occ_m)), 2, sum)

mu.0 <- with(psid, table(occ_f, occ_m))
alpha <- matrix(rep(0, 25), 5, 5)
for (i in 1:5) for (j in 1:5) alpha[i,j] <- mu.0[i,j]*(N.male.0[i]+N.female.0[j])/(N.male.0[i]*N.female.0[j])

random.0 <- matrix(rep(0,25), 5, 5)
for (i in 1:5) for (j in 1:5) random.0[i,j] <- N.male.0[i]*N.female.0[j]/sum(N.male.0)

endogamous.0 <- diag(pmin(N.male.0, N.female.0))

# Table 7 Two-sex SRE

mobility.list.son <- list(mobility.samesex.son, mobility.2sex.son, mobility.perfect.son)
```

```
mobility.list.daughter <- list(mobility.samesex.daughter, mobility.2sex.daughter, mobility.perfect.daughter)

mating.list <- list(random.0, endogamous.0, mu.0)

TSRE.ratio <- rep(0,9)
TSRE.diff <- rep(0,9)

count = 1
for (x in 1:3) {
    for (y in 1:3) {

           new.mobility.son <- matrix(0, 25, 125)
           new.mobility.daughter <- matrix(0, 25, 125)

         for (i in 1:25) {
             new.mobility.son[i, ((i-1)*5+1):(i*5)] <- mobility.list.son[[y]][i,]
             new.mobility.daughter[i, ((i-1)*5+1):(i*5)] <- mobility.list.daughter[[y]][i,]
         }

        G1.son <- t((as.vector(t(mating.list[[x]]))* GRR.son)) %*% new.mobility.son
        G1.daughter <- t((as.vector(t(mating.list[[x]]))* GRR.daughter)) %*% new.mobility.daughter

        TSRE.ratio[count] <- (sum(G1.son[,1]+G1.daughter[,1])/(N.male.0[1]+N.female.0[1])/2)/(sum(G1.son[,((7-1)*5+1)]+G1.daughter[,((7-1)*5+1)])/(N.male.0[2]+N.female.0[2])/2)
        TSRE.diff[count] <- (sum(G1.son[,1]+G1.daughter[,1])/(N.male.0[1]+N.female.0[1])/2)-(sum(G1.son[,((7-1)*5+1)]+G1.daughter[,((7-1)*5+1)])/(N.male.0[2]+N.female.0[2])/2)

        count <- count + 1
        }
  }
}
```