Bayesian estimation and mortality forecast in small areas of Brazil between 2010 and 2030

Marcos R. Gonzaga
Universidade Federal do Rio Grande do Norte
mrcs.roberto@gmail.com

Bernardo L. Queiroz
Universidade Federal de Minas Gerais
lanza@cedeplar.ufmg.br

Everton E. C. Lima
Universidade Estadual de Campinas
evertone@unicamp.br

Flávio H. M. A. Freire
Universidade Federal do Rio Grande do Norte
fhfreire@ccet.ufrn.br

José H. C. Monteiro da Silva
zecosta.monteiro@gmail.com

Abstract

Reliable estimates of mortality rates by sex and age are important for planning, evaluating and allocating public health resources. This demand is become even bigger at more disaggregated geographic levels. However, the accuracy of estimates of mortality rates by sex and age in sub-national populations, especially in developing countries, is still a challenge for demographers and population and health scholars, especially in the context of incomplete vital records and coverage differentiated by age. To deal with this limitation, we propose new methodological alternative to obtain estimates and forecast mortality in scenario of defective data. The procedure involves incorporating uncertainties in the estimates and projections of specific mortality rates at sub-national populations. The uncertainties are related to the two main sources of error: low number of people exposed to death and incomplete coverage in vital records. We use a Bayesian model to produce probabilistic estimates of mortality rates by sex and simple ages for the municipalities of Brazil in 2010. Afterwards, we apply the original model proposed by Lee and Carter (1992) and an improvement of the model is presented to project these rates, making it possible to estimate municipal mortality tables in Brazil for the period 2010-2030. Our results show that the methodological options used were effective in achieving the proposed objectives.

Introduction

Estimates and projections of mortality rates for small areas in Brazil are a challenge for demographers. This problem is even bigger while we are dealing with small locations. Due to the fact that less populated areas are facing usually problems with incomplete coverage of deaths counts, and age exaggeration in both vital records and population data (Luy, 2010; United Nations, 2002).

In the same fashion, mathematical formulations and demographic models are essential for estimating and analyzing trends in demographic components and, consequently, changes in the age structure and size of a population over time. In many cases, indirect demographic methods are also employed to estimate mortality rates and to produce reasonable estimates, even in populations where the quality of census information and vital records are from poor quality (Brass, 1975; Preston et al., 1980; Benneth and Horiuchi, 1981; Hill, 1987). Usually, these methods are based on rigid assumptions of population stability and that is difficult to fulfill in a context of rapid change in population's age structure as we see in Brazil, and greater geographical breakdown of analysis. Moreover, such methods do not provide a measure of uncertainty for estimates (Murray et al, 2010). It is undeniable that combining these demographic methods with statistical techniques is extremely useful for measuring the accuracy of demographic rate estimates, especially in populations with defective data.

In developing regions, the uncertainty in estimating demographic rates at sub-national areas is influenced by two main limitations: 1) rare events occurrence in less populated areas and 2) data quality. The first refers to the low number of people exposed to the event of interest and, consequently, the low number of events happening. Assecond, frequent in developing or less developed countries, refers to the quality of population data and incomplete records of vital records.

To address the high variability in estimated demographic rates by sex and age in small areas, recent studies suggest incorporating the spatial configuration of similar or neighboring areas, thereby reducing possible random fluctuations in estimates (Marshall, 1991; Asunción et al., 1998; Asuncion et al., 2005; Divino et al., 2009; Schmertmann et al., 2013, Queiroz et al., 2013; Gonzaga et al., 2014). In this case, it is assumed that between neighboring areas there is some homogeneity for the occurrence of certain events. The proximity may be purely geographical or any other attributes that shape homogeneity.

As common employed methods, the Bayesian spatial approach has been widely used thanks to new computational technologies and the developments of advanced sampling algorithms. The production of articles in this area began to grow only in the 1990s. In Brazil, for example, the application of this approach to demography has generally occurred in modeling fertility rates for municipalities (Asunción et al., 1998; Schmertmann et al., 2013).

In the case of small-area mortality studies, proposed statistical and mathematical methods for smoothing and projecting mortality rates by age could be also important to reduce the effects of high variability on estimated rates (Heligman and Pollard, 1980; Eilers and Marx, 1996; Pletcher, 1999; Macnab and Dean, 2001; De Beer, 2012). However, these methods are not effective to produce estimates of specific mortality rates in areas with less populated, where recorded death is still a rare event at many ages. On the other hand, traditional demographic methods for estimating the degree of death coverage have limited applicability in the context of rapid demographic transition and consequent non-stable population's age structure (Bhat, 2002; Hill and Queiroz, 2004; Bignami, 2005; Murray et al. ., 2010). The so-called Death Distribution Methods (DDM) were developed based on the demographic equilibrium equation to assess death coverage in relation to the age-listed population in the demographic censuses (Brass, 1975; Preston and Hill, 1980; Preston et al., 1980).They were developed based on strong assumptions of stable population's age structure. Other recent versions of DDMs allow to bypass the stability condition, but still depend on some assumptions that are difficult to verify in small areas, whose age structure is strongly affected by changes in other demographic components, especially due to migration flows or errors of population coverage by age in the demographic censuses (Benneth and Horiuchi, 1981, 1984; Hill, 1987; Bhat, 2002; Hill, 2001; Hill and Choi, 2004; Hill and Queiroz, 2004; Hill, Choi and Timaeus, 2005; Bignami, 2005; Queiroz et al., 2013). In addition to their limited applicability, these methods cannot estimate differences in the degree of coverage by age and do not allow measurement uncertainties of the estimates (Murray et al., 2010).

Recent studies on mortality in small areas in Brazil suggest as way to contour these problems a combination of Death Distribution Methods with indirect standardization techniques to stabilize rates and estimate the degree of coverage of adult mortality in small areas in Brazil since the 1980s (Lima et al. 2014; Queiroz et al. al., 2013; Freire et al., 2015). Other possibility is to apply a shrinkage estimator of the empirical Bayesian estimator class proposed by Marshall (1991), which was adapted and applied to smooth and correct death counts data, yielding estimates of specific adult mortality rates for all Brazilian municipalities (Queiroz et al., 2013; Freire et al., 2015).However, the methods employed did not allow measuring the uncertainties related to the two above cited sources of errors: low number of events and incomplete coverage of vital records.

In this paper, we propose a combination of statistical and demographic models for estimates and projections of specific mortality rates for all microregions and municipalities in Brazil between 2010 and 2030. The employed models provide the opportunity to measure uncertainty in estimates by incorporating researcher's hypotheses and prior knowledge of trends in the level of mortality and data quality of the populations under study. In addition, a flexible model is proposed that incorporates aspects related to the level and structure of mortality at any geographical level of analysis. The following is a recent proposal to combine Bayesian and demographic methods for estimating specific mortality and life expectancy rates for small areas in Brazil. First, we present a statistical method developed by Gonzaga &Schmertmann (2016) to estimate and smooth mortality rates by sex and age in small areas. It is a relational model superior to the indirect standardization demographic technique when the objective is to determine the mortality pattern for small areas with complete death coverage. The following describes a Bayesian model developed to incorporate in the estimates of mortality rates by sex and age of smaller areas, the uncertainties due to the high variability and incomplete coverage of deaths by sex and age.

Initially, we present the Bayesian model for estimating mortality rates by sex and age in all microregions and municipalities of Brazil in 2010. Then after, we illustrate the procedures adopted for forecast mortality rates by sex and age in both the geographical units. Such model allows to incorporate uncertainties related to future trends in the risk of death by sex and simple age of the populations under study. The following are the hypotheses, decisions and procedures made to obtain the estimates according to the limitations found.

Data and Methods

Data sets

We use a combination of datasets (publicly available) that has data on deaths disaggregated by age, sex, cause of death and areas of interest are provided by the Ministry of Health Mortality Information System (SIM/Datasus) 1. Population data are available from the Brazilian Institute of Geography and Statistics (IBGE) 2. Additional information on death data qualities and specific mortality rates in small areas in Brazil were provided by the following research projects.

1. Case-finding death search in the Northeast and Legal Amazon (Szwarcwald et al., 2011). Results available in: http://svs.aids.gov.br/dashboard/buscaAtiva/buscaAtiva.show.mtw;
2. Research results provided by: "Smoothing, estimating and projecting mortality rates in small areas in Brazil". R codes and results available at: http://topals-mortality.schmert.net/ and http://mortality-subregistration.schmert.net/;
3. Research Project: "Estimates of Mortality and Tables Construction for Small Areas in Brazil, 1980-2010" (470866/2014: MCTI / CNPQ / MEC / CAPES / Applied Social Sciences and 454223 / 2014-5: MCTI / CNPQ /Universal). Data availableuponrequestfromresearchers;
4. Other official publications on estimates of death coverage at different geographical levels: Gadelha et al (2002); Albuquerque and Senna (2003); Augustine (2009); Queiroz et al (2017); Paes (2005); GBD (2018), IBGE / MS (2018).

Methods

Estimating and smoothing specific mortality rates for small areas

We estimate mortality rates by sex and age with using the TOPALS relational model proposed by Gonzaga and Schmertmann (2016), as variant of the original one developed by De Beer (2012).

In a TOPALS model, the log mortality schedule is a sum of two functions, a constant schedule (standard) that incorporates basic age patterns and a parametric, piecewise-linear function made up of straight-line segments between designated ages (these ages are called knots) that represents differences between the standard and the log mortality schedule in the population of interest:

$$\underset{101x1}{\lambda(\alpha)} = \underset{101x1}{\lambda^{*}} + \underset{101x7}{\mathbf{B}}\ \underset{7x1}{\alpha} \qquad (1)$$

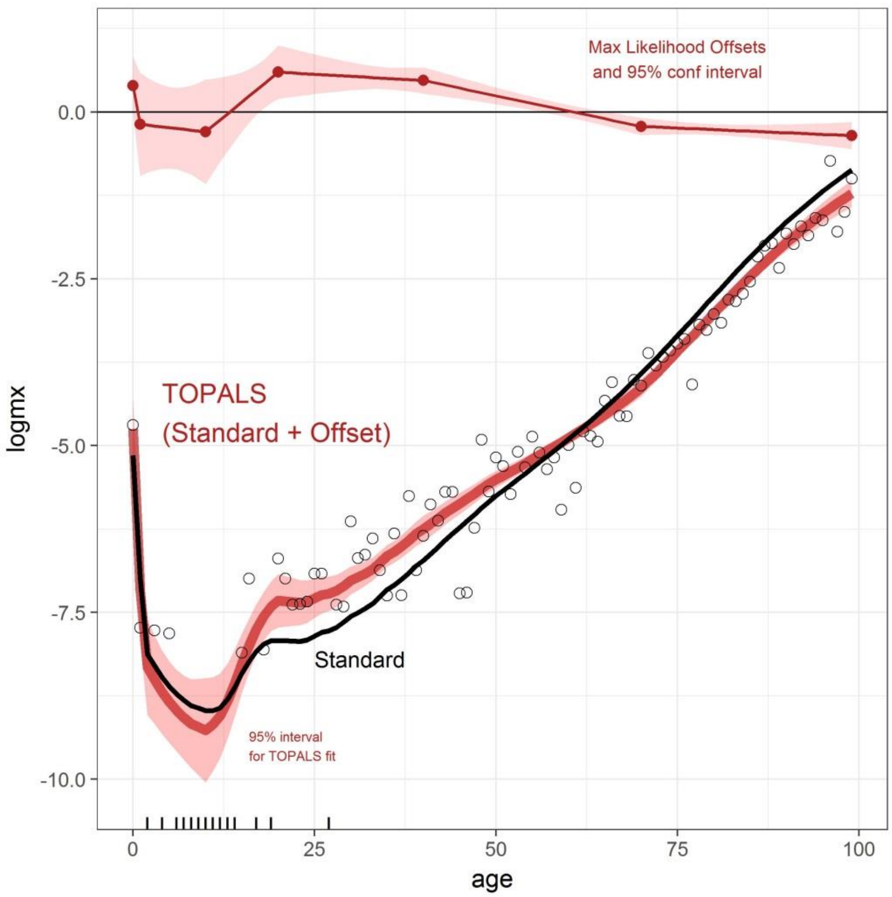
Where:

λ is a vector of log mortality rates in a mesoregion; λ* is a standard schedule (the national log mortality rates);B is a matrix of constants in which each column is a linear B-splines basis function, where knots are defined at exact ages (x = 0, 1, 10, 20, 40, 70, 100) as suggested by Gonzaga and Schmertmann(2016); α is a vector of parameters representing offsets to the standard schedule. In the equation (1) the α values represent additive offsets (λ_x-λ_x^*) to the log mortality rate schedule at knots and offsets change linearly with age between these knots. According to Gonzaga and Schmertmann (2016), based on any set of observed age-specific deaths (Dx) and populations (N_x), deaths are distributed as independent Poisson variables and the log likelihood is:

$$\log L(\alpha) = \sum_{x} \left[D_x \, \lambda_x(\alpha) - N_x \, \exp(\lambda_x(\alpha)) \right] - \sum_{k=0}^{5} \left(\alpha_{k+1} - \alpha_k\right)^2 \qquad (2)$$

The TOPALS method's properties makes the specific choice of a standard schedule far less important than in other relational models used in demography as indirect standardization technique (Gonzaga and Schmertmann, 2016), for example. In fact, as shown by Gonzaga and Schmertmann(2016), in some Brazilian small areas we cannot rely on the assumption that all mesoregions within any state have the same age-specific mortality rates schedule. Finally, since the TOPALS method estimate parameters by maximizing a Poisson likelihood function for age-specific deaths, conditional on age-specific exposure, it give us a 95% interval confidence for the estimates.

Figure 1: TOPALS regression to estimate female mortality rates by age, Microregion of Pará de Minas-MG, Brazil, 2010.



Bayesian model for estimating mortality rates and life tables in small areas in Brazil

Schmertmann& Gonzaga (2018) have developed a Bayesian model that allows a probabilistic assessment of mortality rates and life expectancy in small areas as a function of two sources of errors: high variability in observed rates and incomplete coverage of deaths by age. The integrated Bayesian method uses information on differential in under-registration of death counts at the municipal level, provided by the Case-Finding Search Project (Szwarcwald et al., 2011), and at the state level, provided by Queiroz (2012), Queiroz et al (2013); Freire et al (2015) and IBGE (2013).

For each area of interest, there are a number of exposed persons Nx and a total number of deaths recorded Dx at each age x. In the hypothesis of underreporting of deaths, we have that the true number of deaths (Dx *) is not observed, i.e. Dx* is greater than or equal to Dx (number of deaths recorded). The true number of deaths is assumed to follow an independent Poisson distribution at each age, which depends on an age-mortality rate mortality by age μ_x:

$$D_x^* \sim Poisson(N_x \mu_x) \tag{3}$$

It is also assumed that each death is independently recorded with π_x probability at each age, so that the total death recorded at each age follows a binomial distribution:

$$D_x \sim Binomial(D_x^* \pi_x) \tag{4}$$

Under these circumstances, Schmertmann& Gonzaga (2018) demonstrate that the number of recorded deaths (Dx), implicit by (3) and (4), follows a Poisson distribution:

$$D_x \sim Poisson(N_x \mu_x \pi_x) \tag{5}$$

The model integrates two priori distributions: one on the pattern of age-related mortality rates and the other on the likelihood of death in a given age range.

The priori distribution was used for the registration of death in the first year of life, a Beta distribution and the average is given by the estimate of the degree of coverage of child deaths at the municipal level, estimates provided by the project “Case-finding death search in the Northeast and Legal Amazon ”(Szwarcwald et al., 2011).For deaths over 30 years old, the estimates of the degree of coverage of adult deaths are produced by the Methods of Distribution of Deaths at the state level of different sources were used (Queiroz, 2012; Queiroz et al., 2013; Freire et al. , 2015; IBGE, 2013). Such estimates differ substantially in some cases, especially for states in the North and Northeast regions, revealing the degree of uncertainty about the true degree of coverage of adult deaths in each state of Brazil. With these estimates, a Beta distribution was assumed where the mean and precision parameters are such that the lowest and highest values of the four estimates of degree of coverage published in each state represent the 5th and 95th percentile of the distribution.

In this fashion, the results from the proposed model incorporates the sources of uncertainty in age mortality rates into a statistical perspective, thus not ignoring Demographer's experience and knowledge of the possible range for the degree of coverage translated by Demographic Methods.

Probabilistic model for projecting specific mortality rates for small areas

The method combines a demographic model with time-series method of forecasting. The method involves modeling two factors age and time, and uses matrix decomposition to extract a single time-varying index of mortality rates, which is then forecast using a time-series model. The Lee- Carter method has been considered a powerful method to forecast mortality due to its precision and simple way to model age distribution of death rates [Lee, 2000, Lee and Miller, 2001, Booth and Tickle, 2008]. The more linear trends in age-specific rates, the more robust is the method [Lee, 2000, Lee and Miller, 2001, Booth and Tickle, 2008]. There are a series of variations to the Lee-Carter model, but we used the more traditional one.

To estimate the model, we need a matrix of mx, labor force participation rates and find a solution for Equation 1:

$$Log(m_x) = a_x + b_x + \varepsilon_{x,} \qquad (6)$$

where, a_x and b_x are parameters to be estimated and ε is a set of random disturbances. The solution of this regression is made by applying the Singular Value-Decomposition approach (SVD) on the log of the historical rates matrix. In the model, ax represents the average age pattern of the mortality, b_x represents the amount of mortality change at a given age for a unit of yearly mortality change and k_t measures the general level of mortality.

The Lee-Carter Method requires a series of sex and age-specific log mortality rates to be used in a time series model to forecast the global level of mortality (the k parameter on equation 6). Since we have municipality's estimates only for 2010 census years, we applied the Lee-Carter method on a long time series of log mortality rates for larger regions in Brazil. Then, we used the larger regions' estimated parameters of the method in order to forecast log mortality rates in all municipalities within each respective larger region. The assumption used in this approach is a convergence between levels and age pattern of log mortality rates for all municipalities in each larger region.

Preliminary Results

Figures 2 to 6 below present the preliminary results for the small areas mortality forecast. Figure 2 shows the Males log mortality rates by age for selected municipalities across different regions in Brazil at 2010. Municipalities were select according to different population size (N) and the results shows the powerful of the Schmertmann& Gonzaga (2018) Bayesian method to estimate sex and age-specific mortality rates for larger and small areas, with or without defective vital records across Brazilian regions. It is clear that the smaller exposition the higher the uncertainty about mortality rates by single age, especially in ages where death is a rare event.

The quality of the registry of deaths in Brazil has presented significant improvements in recent years, with coverage of the death registry, for both men and women, above 95%. However, there is still great regional variation. States located in the South and Southeast regions have records of 100% of deaths, for both men and women. In some states in the Northeast and North, the quality of information is lower, but these have recent significant advances when compared to the period 1991 and 2000. In 2010, all the federative units of the South and Southeast regions, as well as some federative units of the Northeast and Central West, presented complete coverage of the death registry. In addition, there was a great improvement in the quality of mortality information in the poorer federative units of the Northeast and North regions, especially those that had the worst record quality in previous periods.

Figure 3 shows estimates of life expectancy at birth e(0) for sex and Brazilian microregions[1] in 2010. The right column shows e(0) estimates without any correction due to under counts of deaths in the vital records. The left column shows e(0) Bayesian estimates. One can see the effect of corrections due to under count of deaths especially in North and Northeast small areas.

Based on 2010 estimates of sex and age-specific mortality rates for all 5.565 municipalities in Brazil, we apply the Lee-Carter method to forecast mortality rates from 2010 to 2030. Since we do not have a time series of mortality rates for municipalities in Brazil, we use the Lee-Carter model parameters estimated from a long time series of mortality rates for larger regions under the assumption of a convergence of levels and age patterns of mortality rates across municipalities within the same larger region. The long time series and forecast of males' mortality rates by larger region in Brazil are presented in the figure 3. In all 5 larger regions one can see the adult over mortality rates due to external causes of deaths, especially homicide and transit accidents (Borges et al, 2012).

Figure 4 present the estimates and forecast of Males log mortality rates by age and larger regions, Brazil (2010-2030). The estimated Lee-Carter model parameter base on trends and forecasting of mortality rates as presented in the figure 4 were used to perform the projection of sex and age-specific mortality rates for all 5.565 municipalities in Brazil from 2010 to 2030. Results for selected municipalities (the same municipalities presented in the figure 2) are showed in the figure 5. One can see the persistent over mortality rates even for the end of the period of projections in all selected municipalities. Finally, the evolution of life expectancy at birth by sex and municipality from 2010 to 2030 are presented in the figure 6.

Next Steps and Expected Results

1. Estimate of log mortality rates and life tables by sex and small areas in Brazil from 1980 to 2010

[1] In Brazil, five larger regions are divided into Federal States (27 total). States are subdivided into mesoregions (137 total), mesoregions into microregions (558), and microregions into municipalities (5,565). Municipalities are the smallest areas responsible for registering vital events.

2. Analyze trends in mortality by regions over time and make a comparison with the results presented here. This comparison can be useful to show how plausible is the approach of using Lee-Carte parameters based on larger regions to perform mortality projection for small areas.
3. Spatial and temporal analysis of mortality in Brazil from 1980 to 2030.

Figure 2 - Males log mortality rates by age for selected municipalities, Brazil (2010)

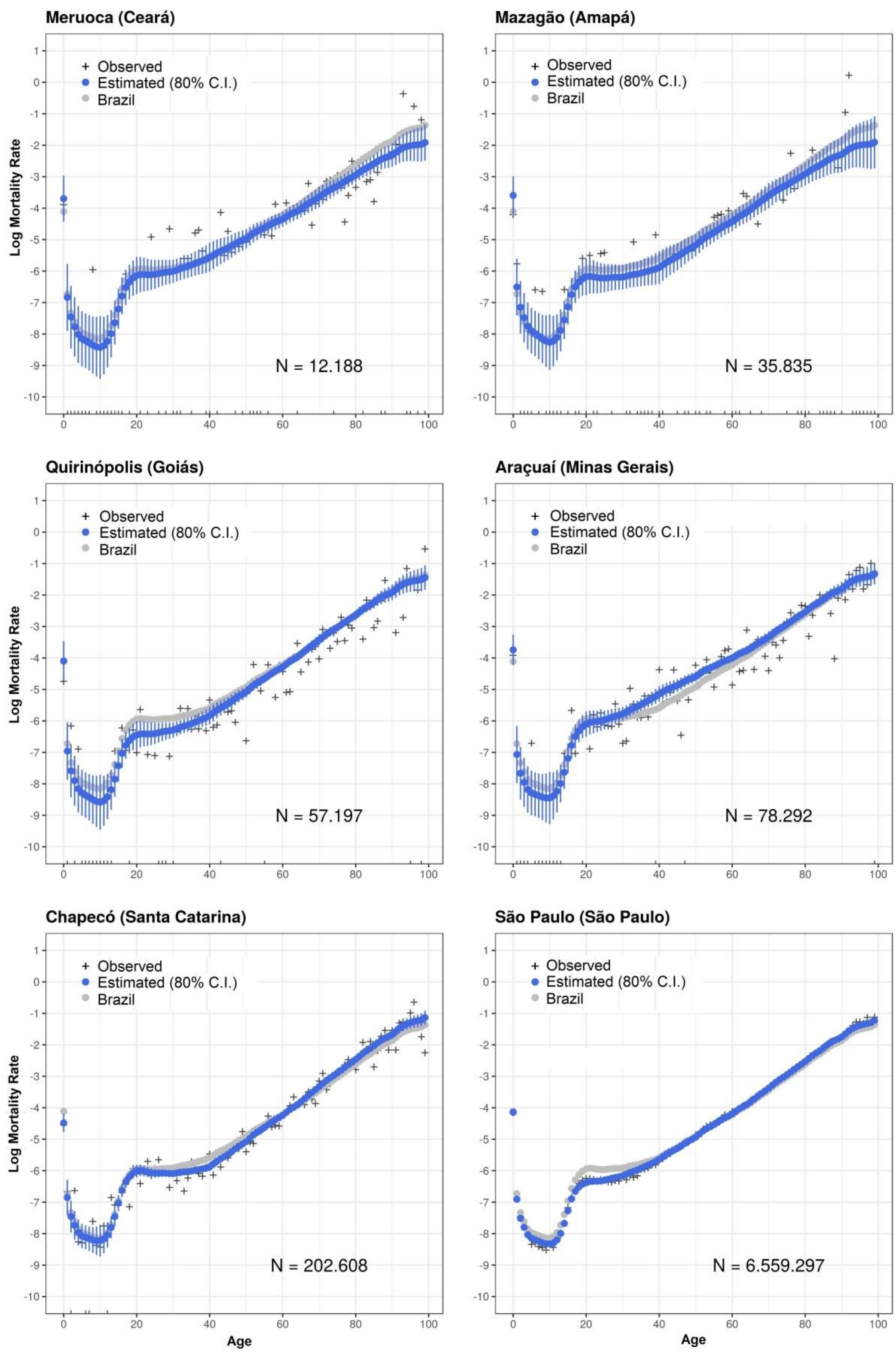


Figure 3 - Life expectancy at birth e(0) for microregions, Brazil (2010)

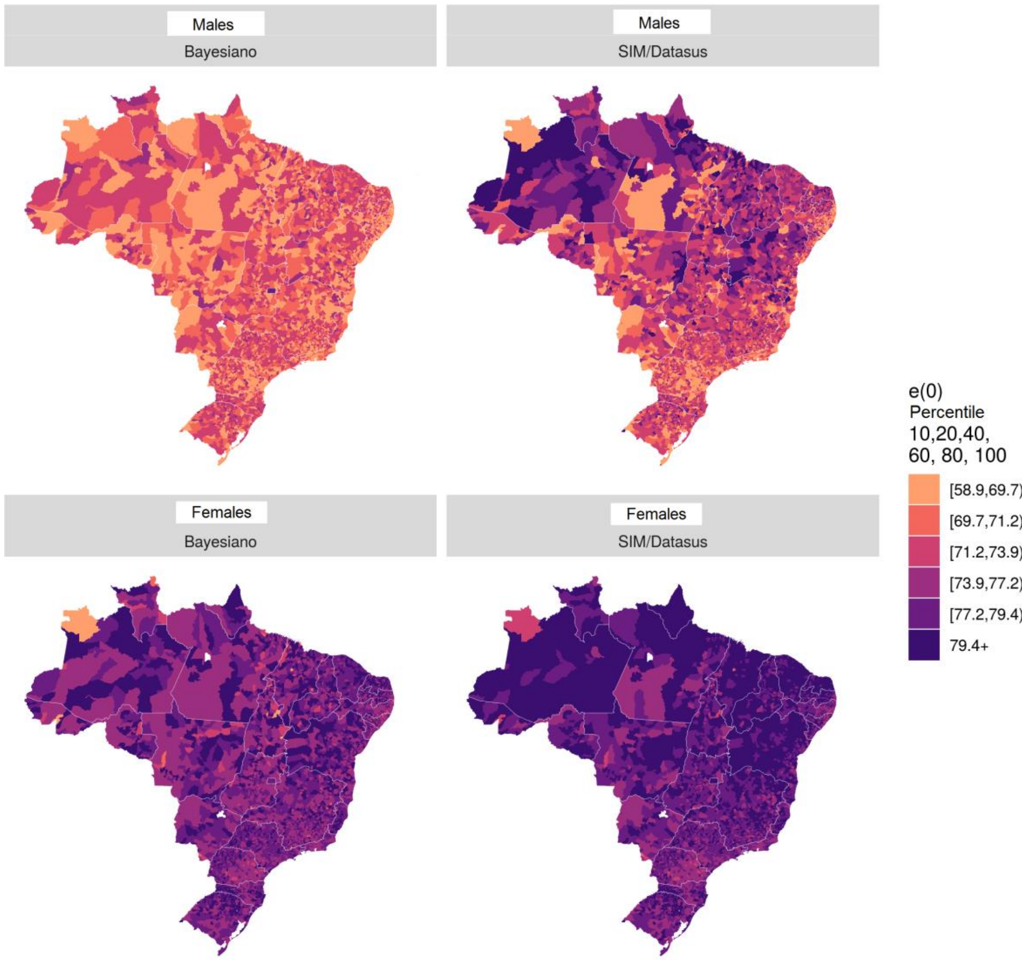


Figure 4–Males log mortality rates for larger regions, Brazil (2010-2030)

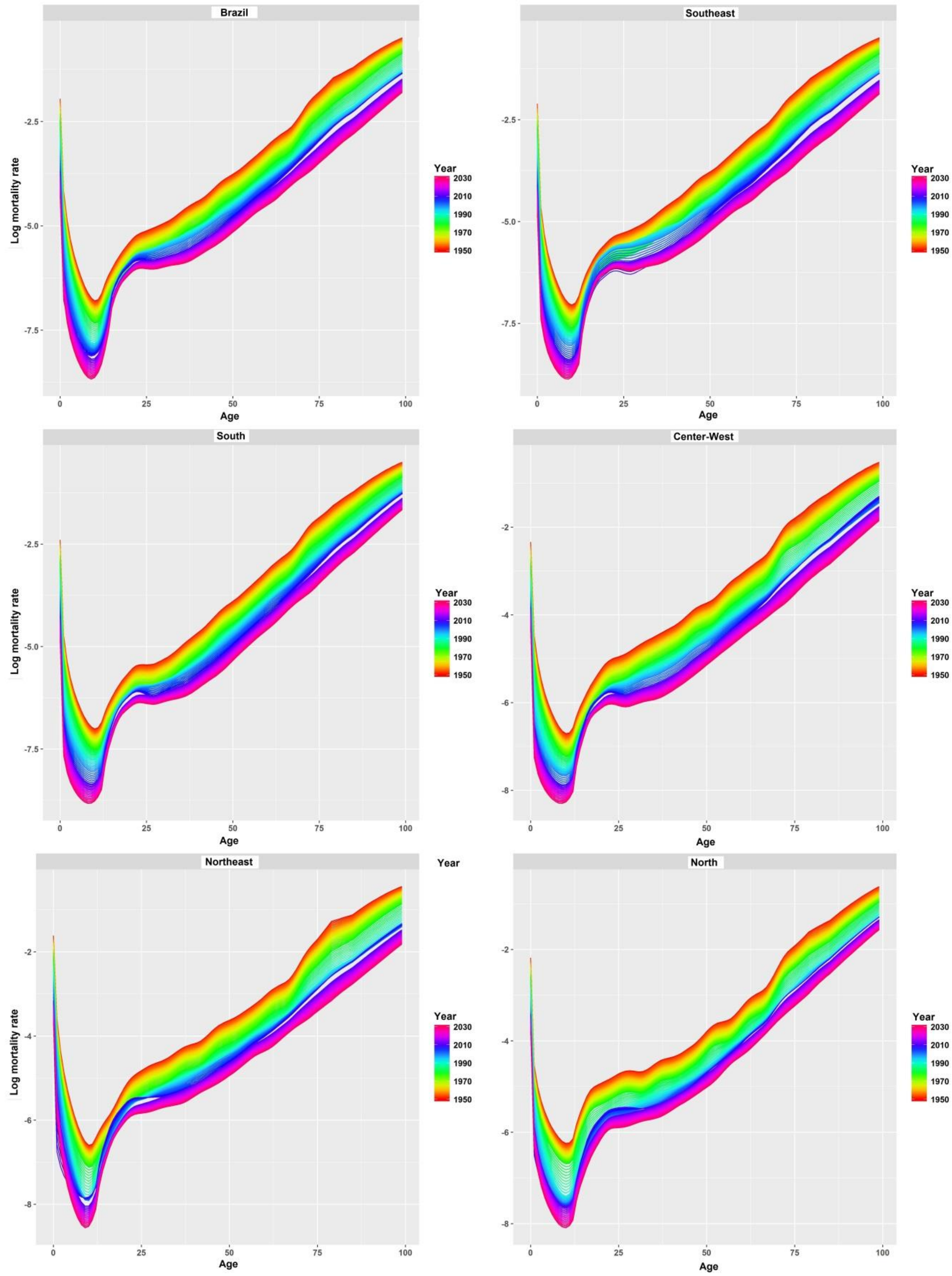


Figure 5–Males log mortality rates for selected municipalities, Brazil (2010-2030)

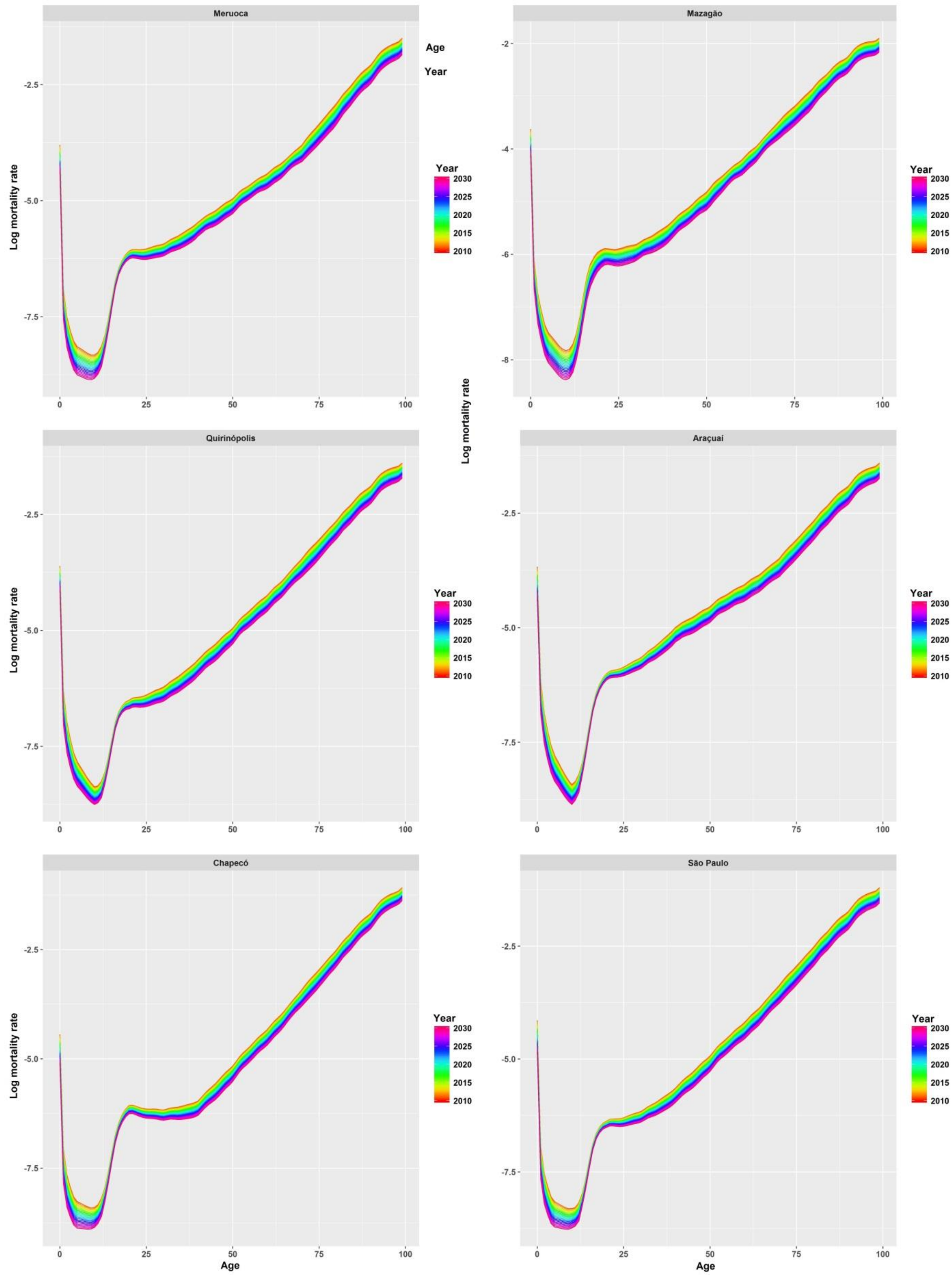
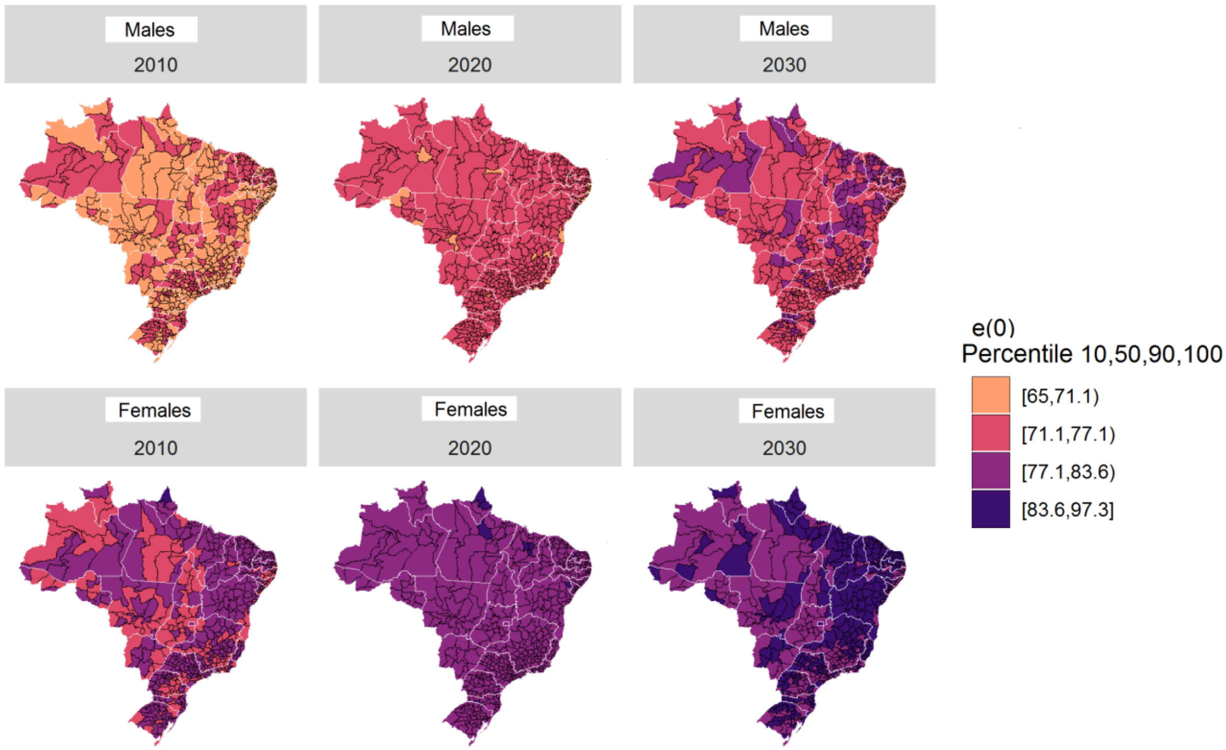


Figure 6–Evolution of Life expectancy at birthby sex across municipalities in Brazil (2010-2030)



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